Chapter 12

Probability and Statistics

What You'll Learn

- **Lessons 12-1 and 12-2** Solve problems involving independent events, dependent events, permutations, and combinations.
- **Lessons 12-3, 12-4, 12-5, and 12-8** Find probability and odds.
- **Lesson 12-6** Find statistical measures.
- **Lesson 12-7** Use the normal distribution.
- **Lesson 12-9** Determine whether a sample is unbiased.

Key Vocabulary

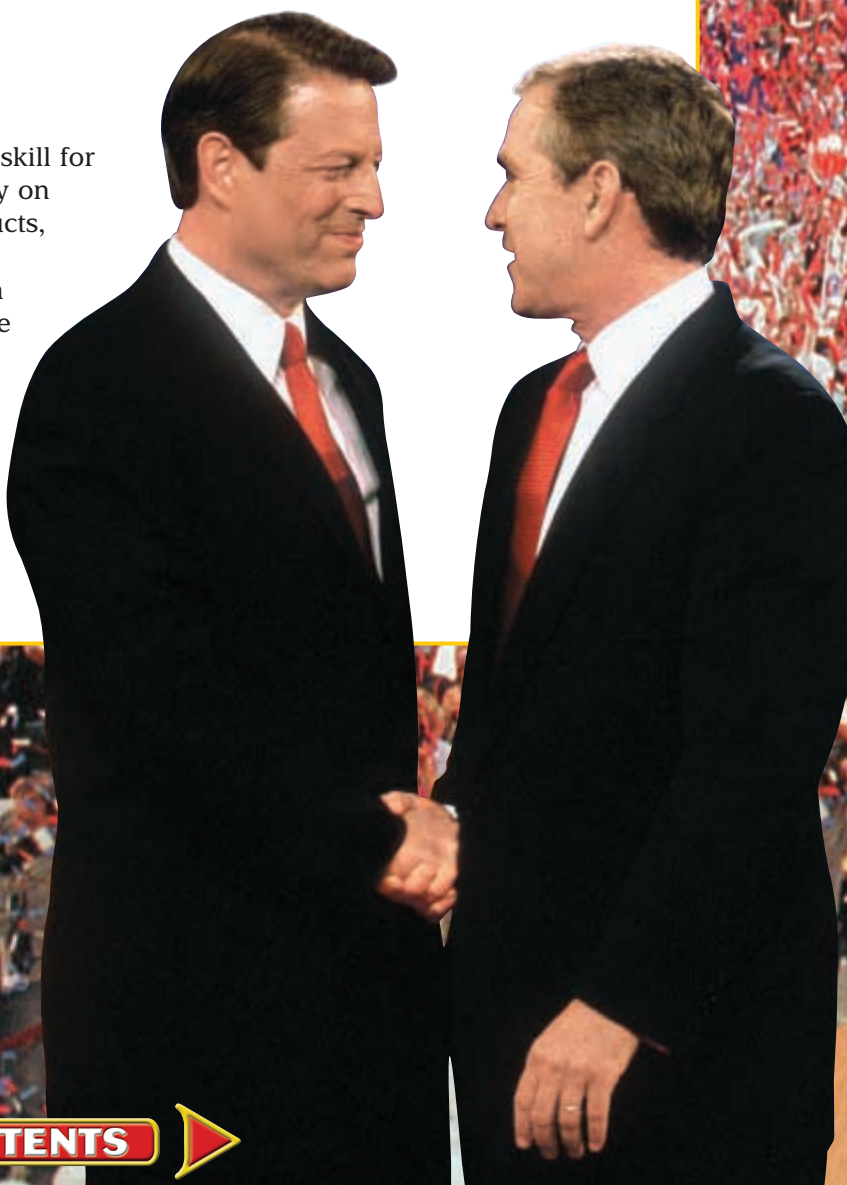
- permutation (p. 638)
- combination (p. 640)
- probability (p. 644)
- measures of central tendency (p. 664)
- measures of variation (p. 665)

Why It's Important

Being able to analyze data is an important skill for every citizen. Business decision-makers rely on statistical measures to ensure quality products, medical researchers test and design new treatments by performing experiments with sample populations, and sports coaches use probabilities to design a winning team.

Each day during a presidential election campaign, journalists report the results of public opinion polls. Pollsters must make sure that the sample they choose accurately represents all of the voters.

You will investigate how opinion polls are used in political campaigns in Lesson 12-9.



Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 12.

For Lesson 12-3 — Find Simple Probability

Find each probability if a die is rolled once.

1. $P(2)$ **2.** $P(5)$ **3.** $P(\text{even number})$

4. $P(\text{odd number})$ **5.** $P(\text{numbers less than 5})$ **6.** $P(\text{numbers greater than 1})$

For Lesson 12-6 — Box-and-Whisker Plots

Make a box-and-whisker plot for each set of data. *(For review, see pages 826 and 827.)*

7. {24, 32, 38, 38, 26, 33, 37, 39, 23, 31, 40, 21}

8. {25, 46, 31, 53, 39, 59, 48, 43, 68, 64, 29}

9. {51, 69, 46, 27, 60, 53, 55, 39, 81, 54, 46, 23}

10. {13.6, 15.1, 14.9, 15.7, 16.0, 14.1, 16.3, 14.3, 13.8}

For Lesson 12-6 — Evaluate Expressions

Evaluate $\sqrt{\frac{(a-b)^2+(c-b)^2}{d}}$ for each set of values. *(For review, see Lesson 5-6.)*

11. $a = 4, b = 7, c = 1, d = 5$ **12.** $a = 2, b = 6, c = 9, d = 5$

13. $a = 5, b = 1, c = 7, d = 4$ **14.** $a = 3, b = 4, c = 11, d = 10$

For Lesson 12-8 — Expand Binomials

Expand each binomial. *(For review, see Lesson 5-2.)*

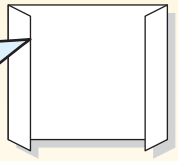
15. $(a + b)^3$ **16.** $(c + d)^4$ **17.** $(m - n)^5$ **18.** $(x + y)^6$

FOLDABLES™ Study Organizer

Make this Foldable to help you organize information about probability and statistics. Begin with one sheet of 11" by 17" paper.

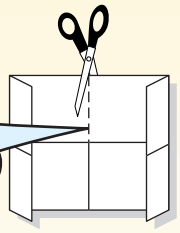
Step 1 Fold

Fold 2" tabs on each of the short sides.



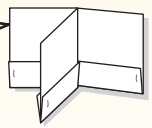
Step 2 Fold and Cut

Then fold in half in both directions. Open and cut as shown.



Step 3 Staple and Label

Refold along the width. Staple each pocket. Label pockets as *The Counting Principle*, *Permutations and Combinations*, *Probability*, and *Statistics*.



Reading and Writing As you read and study the chapter, you can write notes and examples on index cards and store the cards in the Foldable pockets.

12-1 The Counting Principle

What You'll Learn

- Solve problems involving independent events.
- Solve problems involving dependent events.

Vocabulary

- outcomes
- sample space
- event
- independent events
- Fundamental Counting Principle
- dependent events

How can you count the maximum number of license plates a state can issue?

Most states have letters and digits on their license plates. The number of possible plates is too great to count by listing all of the possibilities. It is much more efficient to count the number of possibilities by using the Fundamental Counting Principle.

INDEPENDENT EVENTS An **outcome** is the result of a single trial. For example, the trial of flipping a coin once has two outcomes: head or tail. The set of all possible outcomes is called the **sample space**. An **event** consists of one or more outcomes of a trial. The choices of letters and digits to be put on a license plate are called **independent events** because each letter or digit chosen does *not* affect the choices for the others.

For situations in which the number of choices leads to a small number of total possibilities, you can use a tree diagram or a table to count them.

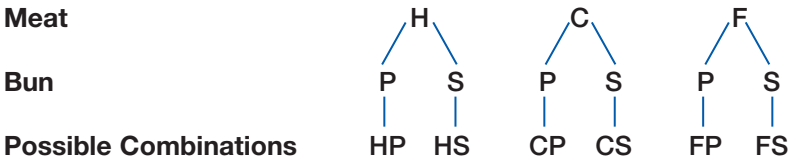
Example 1 Independent Events

FOOD **A sandwich cart offers customers a choice of hamburger, chicken, or fish on either a plain or a sesame seed bun. How many different combinations of meat and a bun are possible?**

First, note that the choice of the type of meat does not affect the choice of the type of bun, so these events are independent.

Method 1 Tree Diagram

Let H represent hamburger, C, chicken, F, fish, P, plain, and S, sesame seed. Make a tree diagram in which the first row shows the choice of meat and the second row shows the choice of bun.



There are six possible outcomes.

Method 2 Make a Table

Make a table in which each row represents a type of meat and each column represents a type of bun.

This method also shows that there are six outcomes.

		Bun	
		Plain	Sesame
Meat	Hamburger	HP	HS
	Chicken	CP	CS
	Fish	FP	FS

CONTENTS

Notice that in Example 1, there are 3 ways to choose the type of meat, 2 ways to choose the type of bun, and 3 · 2 or 6 total ways to choose a combination of the two. This illustrates the **Fundamental Counting Principle**.

Key Concept — Fundamental Counting Principle

- **Words** If event M can occur in m ways and is followed by event N that can occur in n ways, then event M followed by event N can occur in $m \cdot n$ ways.
- **Example** If event M can occur in 2 ways and event N can occur in 3 ways, then M followed by N can occur in $2 \cdot 3$ or 6 ways.

This rule can be extended to any number of events.

Standardized Test Practice — Example 2 Fundamental Counting Principle

Multiple-Choice Test Item

Kim won a contest on a radio station. The prize was a restaurant gift certificate and tickets to a sporting event. She can select one of three different restaurants and tickets to a football, baseball, basketball, or hockey game. How many different ways can she select a restaurant followed by a sporting event?

(A) 7 (B) 12 (C) 15 (D) 16

Read the Test Item

Her choice of a restaurant does not affect her choice of a sporting event, so these events are independent.

Solve the Test Item

There are 3 ways she can choose a restaurant and there are 4 ways she can choose the sporting event. By the Fundamental Counting Principle, there are $3 \cdot 4$ or 12 total ways she can choose her two prizes. The answer is B.

The Princeton Review Test-Taking Tip

Remember that you can check your answer by making a tree diagram or a table showing the outcomes.

The Fundamental Counting Principle can be used to count the number of outcomes possible for any number of successive events.

Example 3 More than Two Independent Events

COMMUNICATION **Many answering machines allow owners to call home and get their messages by entering a 3-digit code. How many codes are possible?**

The choice of any digit does not affect the other two digits, so the choices of the digits are independent events.

There are 10 possible first digits in the code, 10 possible second digits, and 10 possible third digits. So, there are $10 \cdot 10 \cdot 10$ or 1000 possible different code numbers.

Study Tip

Reading Math

Independent and *dependent* have the same meaning in mathematics as they do in ordinary language.

DEPENDENT EVENTS Some situations involve dependent events. With **dependent events**, the outcome of one event *does* affect the outcome of another event. The Fundamental Counting Principle applies to dependent events as well as independent events.

Example 4 Dependent Events

SCHOOL **Charlita wants to take 6 different classes next year. Assuming that each class is offered each period, how many different schedules could she have?**

When Charlita schedules a given class for a given period, she cannot schedule that class for any other period. Therefore, the choices of which class to schedule each period are dependent events.

There are 6 classes Charlita can take during first period. That leaves 5 classes she can take second period. After she chooses which classes to take the first two periods, there are 4 remaining choices for third period, and so on.

Period	1st	2nd	3rd	4th	5th	6th
Number of Choices	6	5	4	3	2	1

There are $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ or 720 schedules that Charlita could have.

Note that $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 6!$.

Study Tip

Look Back

To review **factorials**, see Lesson 11-7.

Concept Summary Independent and Dependent Events

- **Words** If the outcome of an event does *not* affect the outcome of another event, the two events are *independent*.
- **Example** Tossing a coin and rolling a die are independent events.

- **Words** If the outcome of an event *does* affect the outcome of another event, the two events are *dependent*.
- **Example** Taking a piece of candy from a jar and then taking a second piece without replacing the first are dependent events because taking the first piece affects what is available to be taken next.

Check for Understanding

Concept Check

1. **List** the possible outcomes when a coin is tossed three times. Use H for heads and T for tails.
2. **OPEN ENDED** Describe a situation in which you can use the Fundamental Counting Principle to show that there are 18 total possibilities.
3. **Explain** how choosing to buy a car or a pickup truck and then selecting the color of the vehicle could be dependent events.

Guided Practice

State whether the events are *independent* or *dependent*.

4. choosing the color and size of a pair of shoes
5. choosing the winner and runner-up at a dog show

Solve each problem.

6. An ice cream shop offers a choice of two types of cones and 15 flavors of ice cream. How many different 1-scoop ice cream cones can a customer order?
7. Lance's math quiz has eight true-false questions. How many different choices for giving answers to the eight questions are possible?
8. For a college application, Macawi must select one of five topics on which to write a short essay. She must also select a different topic from the list for a longer essay. How many ways can she choose the topics for the two essays?

Standardized Test Practice

A B C D

9. A bookshelf holds 4 different biographies and 5 different mystery novels. How many ways can one book of each type be selected?

(A) 1 (B) 9 (C) 10 (D) 20

Practice and Apply

Homework Help

For Exercises	See Examples
10–23, 25–27	1–4

Extra Practice

See page 854.

State whether the events are *independent* or *dependent*.

10. choosing a president, vice president, secretary, and treasurer for Student Council, assuming that a person can hold only one office

11. selecting a fiction book and a nonfiction book at the library

12. Each of six people guess the total number of points scored in a basketball game. Each person writes down his or her guess without telling what it is.

13. The letters A through Z are written on pieces of paper and placed in a jar. Four of them are selected one after the other without replacing any of them.

Solve each problem.

14. Tim wants to buy one of three different albums he sees in a music store. Each is available on tape and on CD. From how many combinations of album and format does he have to choose?

15. A video store has 8 new releases this week. Each is available on videotape and on DVD. How many ways can a customer choose a new release and a format to rent?

16. Carlos has homework to do in math, chemistry, and English. How many ways can he choose the order in which to do his homework?

17. The menu for a banquet has a choice of 2 types of salad, 5 main courses, and 3 desserts. How many ways can a salad, main course, and dessert be selected to form a meal?

18. A golf club manufacturer makes drivers with 4 different shaft lengths, 3 different lofts, 2 different grips, and 2 different club head materials. How many different combinations are possible?

19. Each question on a five-question multiple-choice quiz has answer choices labeled A, B, C, and D. How many different ways can a student answer the five questions?

20. How many ways can six different books be arranged on a shelf if one of the books is a dictionary and it must be on an end?

21. In how many orders can eight actors be listed in the opening credits of a movie if the leading actor must be listed first or last?

22. **PASSWORDS** Abby is registering at a Web site. She must select a password containing 6 numerals to be able to use the site. How many passwords are allowed if no digit may be used more than once?

23. **ENTERTAINMENT** Solve the problem in the comic strip below. Assume that the books are all different.

Peanuts®

24. **CRITICAL THINKING** The members of the Math Club need to elect a president and a vice-president. They determine that there are a total of 272 ways that they can fill the positions with two different members. How many people are in the Math Club?

WebQuest

You can use the Fundamental Counting Principle to list possible outcomes in games. Visit www.algebra2.com/webquest to continue work on your WebQuest project.

25. **HOME SECURITY** How many different 5-digit codes are possible using the keypad shown at the right if the first digit cannot be 0 and no digit may be used more than once?

More About. . .

Area Codes

Before 1995, area codes had the following format.

(XYZ)

X = 2, 3, …, or 9

Y = 0 or 1

Z = 0, 1, 2, …, or 9

Source: www.nanpa.com

AREA CODES **For Exercises 26 and 27, refer to the information about telephone area codes at the left.**

26. How many area codes were possible before 1995?

27. In 1995, the restriction on the middle digit was removed, allowing any digit in that position. How many total codes were possible after this change was made?

28. **RESEARCH** Use the Internet or other resource to find the configuration of letters and numbers on license plates in your state. Then find the number of possible plates.

29. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can you count the maximum number of license plates a state can issue?

Include the following in your answer:

- an explanation of how to use the Fundamental Counting Principle to find the number of different license plates in a state such as Florida, which has 3 letters followed by 3 numbers, and
- a way that a state can increase the number of possible plates without increasing the length of the plate number.

Standardized Test Practice

A B C D

30. How many numbers between 100 and 999, inclusive, have 7 in the tens place?

(A) 90 (B) 100 (C) 110 (D) 120

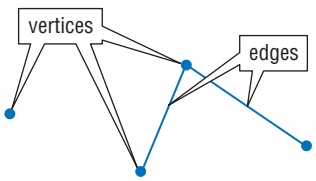
31. A coin is tossed four times. How many possible sequences of heads or tails are possible?

(A) 4 (B) 8 (C) 16 (D) 32

Extending the Lesson

For Exercises 32 and 33, use the following information.

A **finite graph** is a collection of points, called **vertices**, and segments, called **edges**, connecting the vertices. For example, the graph shown at the right has 4 vertices and 2 edges.



32. Suppose a graph has 10 vertices and each pair of vertices is connected by exactly one edge. Find the number of edges in the graph. (*Hint:* If you use the Fundamental Counting Principle, be sure to count each edge only once.)

33. **TRANSPORTATION** The table shows the distances in miles of the roads between some towns. Draw a graph in which the vertices represent the towns and the edges are labeled with the lengths of the roads. Use your graph to find the length of the shortest route from Greenville to Red Rock.

Route	Miles
Greenville to Roseburg	14
Greenville to Bluemont	12
Greenville to Whiteston	9
Roseburg to Bluemont	8
Bluemont to Whiteston	5
Roseburg to Red Rock	7
Bluemont to Red Rock	9
Whiteston to Red Rock	11

Maintain Your Skills

Mixed Review

34. Prove that $4 + 7 + 10 \cdots + (3n + 1) = \frac{n(3n+5)}{2}$ for all positive integers n. *(Lesson 11-8)*

Find the indicated term of each expansion. *(Lesson 11-7)*

35. third term of $(x + y)^8$

36. fifth term of $(2a - b)^7$

Evaluate each expression. *(Lesson 10-2)*

37. $\log_2 128$

38. $\log_3 243$

39. $\log_9 3$

Simplify each expression. *(Lesson 9-1)*

40. $-\frac{x^2 - y^2}{x + y} \cdot \frac{1}{x - y}$

41. $\frac{\frac{x^2}{x^2 - 25y^2}}{\frac{x}{5y - x}}$

42. CARTOGRAPHY Edison is located at (9, 3) in the coordinate system on a road map. Kettering is located at (12, 5) on the same map. Each side of a square on the map represents 10 miles. To the nearest mile, what is the distance between Edison and Kettering? *(Lesson 8-1)*

Solve each equation. *(Lesson 7-3)*

43. $x^4 - 5x^2 + 4 = 0$

44. $y^4 + 4y^3 + 4y^2 = 0$

Write an equation for the parabola with the given vertex that passes through the given point. *(Lesson 6-6)*

45. vertex (3, 2), point (5, 6)

46. vertex (−1, 4), point (−2, 2)

47. vertex (0, 8), point (4, 0)

Solve each equation. *(Lesson 5-8)*

48. $\sqrt{2x + 1} = 3$

49. $3 + \sqrt{x + 1} = 5$

50. $\sqrt{x} + \sqrt{x + 5} = 5$

Find the inverse of each matrix, if it exists. *(Lesson 4-7)*

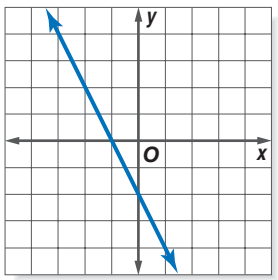
51. $\begin{bmatrix} 3 & 1 \\ -4 & 1 \end{bmatrix}$

52. $\begin{bmatrix} 4 & -5 \\ 2 & -1 \end{bmatrix}$

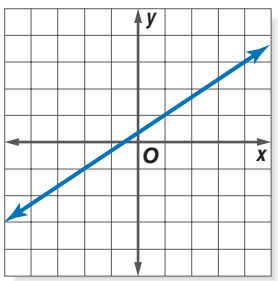
53. $\begin{bmatrix} -3 & 2 \\ -6 & 4 \end{bmatrix}$

Write an equation in slope-intercept form for each graph. *(Lesson 2-4)*

54.



55.



Getting Ready for the Next Lesson

PREREQUISITE SKILL **Evaluate each expression.**

*(To review **factorials**, see Lesson 11-7.)*

56. $\frac{5!}{2!}$

57. $\frac{6!}{4!}$

58. $\frac{7!}{3!}$

59. $\frac{6!}{1!}$

60. $\frac{4!}{2!2!}$

61. $\frac{6!}{2!4!}$

62. $\frac{8!}{3!5!}$

63. $\frac{5!}{5!0!}$

12-2 Permutations and Combinations

What You'll Learn

- Solve problems involving linear permutations.
- Solve problems involving combinations.

How do permutations and combinations apply to softball?

When the manager of a softball team fills out her team's lineup card before the game, the order in which she fills in the names is important because it determines the order in which the players will bat.

Suppose she has 7 possible players in mind for the top 4 spots in the lineup. You know from the Fundamental Counting Principle that there are $7 \cdot 6 \cdot 5 \cdot 4$ or 840 ways that she could assign players to the top 4 spots.

Vocabulary

- permutation
- linear permutation
- combination

PERMUTATIONS When a group of objects or people are arranged in a certain order, the arrangement is called a **permutation**. In a permutation, the *order* of the objects is very important. The arrangement of objects or people in a line is called a **linear permutation**.

Notice that $7 \cdot 6 \cdot 5 \cdot 4$ is the product of the first 4 factors of 7!. You can rewrite this product in terms of 7!.

$$7 \cdot 6 \cdot 5 \cdot 4 = 7 \cdot 6 \cdot 5 \cdot 4 \cdot \frac{3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1}$$ Multiply by $\frac{3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1}$ or 1.

$$= \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} \text{ or } \frac{7!}{3!}$$ $7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ and $3! = 3 \cdot 2 \cdot 1$

Notice that 3! is the same as $(7 - 4)!$.

Study Tip

Reading Math
The expression $P(n, r)$ is read *the number of permutations of n objects taken r at a time*. It is sometimes written as ${}_nP_r$.

The number of ways to arrange 7 people or objects taken 4 at a time is written $P(7, 4)$. The expression for the softball lineup above is a case of the following formula.

Key Concept — **Permutations**

The number of permutations of n distinct objects taken r at a time is given by

$$P(n, r) = \frac{n!}{(n - r)!}.$$

Example 1 Permutation

FIGURE SKATING **There are 10 finalists in a figure skating competition. How many ways can gold, silver, and bronze medals be awarded?**

Since each winner will receive a different medal, order is important. You must find the number of permutations of 10 things taken 3 at a time.

$$P(n, r) = \frac{n!}{(n - r)!}$$ Permutation formula

$$P(10, 3) = \frac{10!}{(10 - 3!)}$$ $n = 10, r = 3$

$$= \frac{10!}{7!}$$ Simplify.

$$= \frac{10 \cdot 9 \cdot 8 \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$$ or 720 Divide by common factors.

The gold, silver, and bronze medals can be awarded in 720 ways.

Notice that in Example 1, all of the factors of $(n - r)!$ are also factors of $n!$. Instead of writing all of the factors, you can also evaluate the expression in the following way.

$$\frac{10!}{(10 - 3)!} = \frac{10!}{7!}$$ Simplify.

$$= \frac{10 \cdot 9 \cdot 8 \cdot 7!}{7!}$$ $\frac{7!}{7!} = 1$

$$= 10 \cdot 9 \cdot 8 \text{ or } 720$$ Multiply.

Suppose you want to rearrange the letters of the word *geometry* to see if you can make a different word. If the two *e*'s were not identical, the eight letters in the word could be arranged in $P(8, 8)$ or 8! ways. To account for the identical *e*'s, divide $P(8, 8)$ or 40,320 by the number of arrangements of *e*. The two *e*'s can be arranged in $P(2, 2)$ or 2! ways.

$$\frac{P(8, 8)}{P(2, 2)} = \frac{8!}{2!}$$ Divide.

$$= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2!}{2!}$$ or 20,160 Simplify.

Thus, there are 20,160 ways to arrange the letters in *geometry*.

When some letters or objects are alike, use the rule below to find the number of permutations.

Key Concept — **Permutations with Repetitions**

The number of permutations of n objects of which p are alike and q are alike is $\frac{n!}{p!q!}$.

This rule can be extended to any number of objects that are repeated.

Example 2 Permutation with Repetition

How many different ways can the letters of the word *MISSISSIPPI* be arranged?

The second, fifth, eighth, and eleventh letters are each I.

The third, fourth, sixth, and seventh letters are each S.

The ninth and tenth letters are each P.

You need to find the number of permutations of 11 letters of which 4 of one letter, 4 of another letter, and 2 of another letter are the same.

$$\frac{11!}{4!4!2!} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4!}{4!4!2!}$$ or 34,650

There are 34,650 ways to arrange the letters.

Study Tip

Permutations and Combinations

- If order in an arrangement *is* important, the arrangement is a *permutation*.
- If order is *not* important, the arrangement is a *combination*.

COMBINATIONS An arrangement or selection of objects in which order is *not* important is called a **combination**. The number of combinations of n objects taken r at a time is written $C(n, r)$. *It is sometimes written ${}_nC_r$.*

You know that there are $P(n, r)$ ways to select r objects from a group of n if the order is important. There are $r!$ ways to order the r objects that are selected, so there are $r!$ permutations that are all the same combination. Therefore,

$$C(n, r) = \frac{P(n, r)}{r!} \text{ or } \frac{n!}{(n - r)!r!}.$$

Key Concept — **Combinations**

The number of combinations of n distinct objects taken r at a time is given by

$$C(n, r) = \frac{n!}{(n - r)!r!}.$$

Example 3 Combination

A group of seven students working on a project needs to choose two from their group to present the group's report to the class. How many ways can they choose the two students?

Since the order they choose the students is not important, you must find the number of combinations of 7 students taken 2 at a time.

$$C(n, r) = \frac{n!}{(n - r)!r!} \quad \text{Combination formula}$$

$$C(7, 2) = \frac{7!}{(7 - 2)!2!} \quad n = 7 \text{ and } r = 2$$

$$= \frac{7!}{5!2!} \text{ or } 21 \quad \text{Simplify.}$$

There are 21 possible ways to choose the two students.

In more complicated situations, you may need to multiply combinations and/or permutations.

Study Tip

Deck of Cards

In this text, a *standard deck of cards* always means a deck of 52 playing cards. There are 4 suits—clubs (black), diamonds (red), hearts (red), and spades (black)—with 13 cards in each suit.

Example 4 Multiple Events

Five cards are drawn from a standard deck of cards. How many hands consist of three clubs and two diamonds?

By the Fundamental Counting Principle, you can multiply the number of ways to select three clubs and the number of ways to select two diamonds.

Only the cards in the hand matter, not the order in which they were drawn, so use combinations.

$C(13, 3)$ Three of 13 clubs are to be drawn.

$C(13, 2)$ Two of 13 diamonds are to be drawn.

$$C(13, 3) \cdot C(13, 2) = \frac{13!}{(13 - 3)!3!} \cdot \frac{13!}{(13 - 2)!2!} \quad \text{Combination formula}$$

$$= \frac{13!}{10!3!} \cdot \frac{13!}{11!2!} \quad \text{Subtract.}$$

$$= 286 \cdot 78 \text{ or } 22{,}308 \quad \text{Simplify.}$$

There are 22,308 hands consisting of 3 clubs and 2 diamonds.

Check for Understanding

Concept Check

1. **OPEN ENDED** Describe a situation in which the number of outcomes is given by $P(6, 3)$.
2. **Show** that $C(n, n - r) = C(n, r)$.
3. **Determine** whether the statement $C(n, r) = P(n, r)$ is *sometimes*, *always*, or *never* true. Explain your reasoning.

Guided Practice

Evaluate each expression.

4. $P(5, 3)$
5. $P(6, 3)$
6. $C(4, 2)$
7. $C(6, 1)$

Determine whether each situation involves a *permutation* or a *combination*. Then find the number of possibilities.

8. choosing 2 different pizza toppings from a list of 6
9. seven shoppers in line at a checkout counter
10. an arrangement of the letters in the word *intercept*

Application

11. **SCHOOL** The principal at Cobb County High School wants to start a mentoring group. He needs to narrow his choice of students to be mentored to six from a group of nine. How many ways can a group of six be selected?

Practice and Apply

Homework Help

For Exercises	See Examples
12–15	1
16–19	3
20, 21, 32–35	4
22–31	1–3

Extra Practice

See page 854.

More About . . .

Languages

The Hawaiian language consists of only twelve letters, the vowels a, e, i, o, and u and the consonants h, k, l, m, n, p, and w.

Source: www.andhawaii.com

Evaluate each expression.

12. $P(8, 2)$
13. $P(9, 1)$
14. $P(7, 5)$
15. $P(12, 6)$
16. $C(5, 2)$
17. $C(8, 4)$
18. $C(12, 7)$
19. $C(10, 4)$
20. $C(12, 4) \cdot C(8, 3)$
21. $C(9, 3) \cdot C(6, 2)$

Determine whether each situation involves a *permutation* or a *combination*. Then find the number of possibilities.

22. the winner and first, second, and third runners-up in a contest with 10 finalists
23. selecting two of eight employees to attend a business seminar
24. an arrangement of the letters in the word *algebra*
25. placing an algebra book, a geometry book, a chemistry book, an English book, and a health book on a shelf
26. selecting nine books to check out of the library from a reading list of twelve
27. an arrangement of the letters in the word *parallel*
28. choosing two CDs to buy from ten that are on sale
29. selecting three of fifteen flavors of ice cream at the grocery store
30. **MOVIES** The manager of a four-screen movie theater is deciding which of 12 available movies to show. The screens are in rooms with different seating capacities. How many ways can he show four different movies on the screens?
31. **LANGUAGES** How many different arrangements of the letters of the Hawaiian word *aloha* are possible?
32. **GOVERNMENT** How many ways can five members of the 100-member United States Senate be chosen to be put on a committee?

33. How many ways can a hand of five cards consisting of four cards from one suit and one card from another suit be drawn from a standard deck of cards?

34. How many ways can a hand of five cards consisting of three cards from one suit and two cards from another suit be drawn from a standard deck of cards?

35. LOTTERIES In a multi-state lottery, the player must guess which five of forty nine white balls numbered from 1 to 49 will be drawn. The order in which the balls are drawn does not matter. The player must also guess which one of forty-two red balls numbered from 1 to 42 will be drawn. How many ways can the player fill out a lottery ticket?

36. CARD GAMES *Hanafuda* is a Japanese game that uses a deck of cards made up of 12 suits, with each suit having four cards. How many 7-card hands can be formed so that 3 are from one suit and 4 are from another?

37. CRITICAL THINKING Show that $C(n - 1, r) + C(n - 1, r - 1) = C(n, r)$.

38. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How do permutations and combinations apply to softball?

Include the following in your answer:

- an explanation of how to find the number of 9-person lineups that are possible, and
- an explanation of how many ways there are to choose 9 players if 16 players show up for a game.

More About . . .

Card Games

Hanafuda cards are often called "flower cards" because each suit is depicted by a different flower. Each flower is representative of the calendar month in which the flower blooms.

Source: www.gamesdomain.com

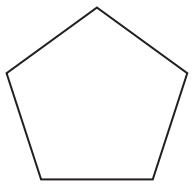
Standardized Test Practice

A B C D

39. How many ways can eight runners in an Olympic race finish in first, second, and third places?

(A) 8 (B) 24 (C) 56 (D) 336

40. How many diagonals can be drawn in the pentagon?

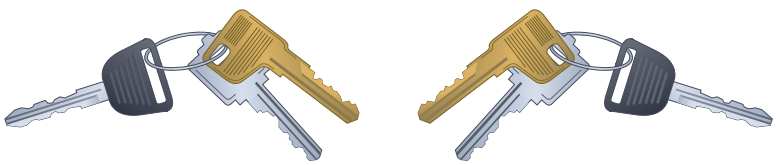


(A) 5 (B) 10

(C) 15 (D) 20

Extending the Lesson

When n distinct objects are arranged in a circle, there are n ways that the arrangement can be rotated to obtain an arrangement that is really the same as the original. For example, the two arrangements of three objects shown below are the same. Therefore, the number of **circular permutations** of n distinct objects is $\frac{n!}{n}$ or $(n - 1)!$ *Note that the keys are not turned over.*



Find the number of possibilities for each situation.

41. a basketball huddle of 5 players

42. four different dishes on a revolving tray in the middle of a table at a Chinese restaurant

43. six quarters with designs from six different states arranged in a circle on top of your desk

Maintain Your Skills

Mixed Review

44. Darius can do his homework in pencil or pen, using lined or unlined paper, and on one or both sides of each page. How many ways can he prepare his homework? *(Lesson 12-1)*

45. A customer in an ice cream shop can order a sundae with a choice of 10 flavors of ice cream, a choice of 4 flavors of sauce, and with or without a cherry on top. How many different sundaes are possible? *(Lesson 12-1)*

Find a counterexample to each statement. *(Lesson 11-8)*

46. $1 + 2 + 3 + \ldots + n = 2n - 1$

47. $5^n + 1$ is divisible by 6.

Solve each equation or inequality. *(Lesson 10-5)*

48. $3e^x + 1 = 2$

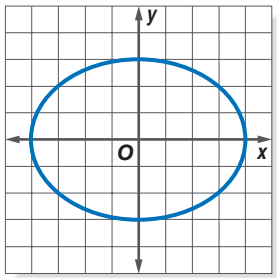
49. $e^{2x} > 5$

50. $\ln (x - 1) = 3$

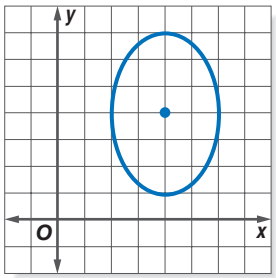
51. CONSTRUCTION A painter works on a job for 10 days and is then joined by an associate. Together they finish the job in 6 more days. The associate could have done the job in 30 days. How long would it have taken the painter to do the job alone? *(Lesson 9-6)*

Write an equation for each ellipse. *(Lesson 8-4)*

52.



53.



Find $p(-1)$ and $p(5)$ for each function. *(Lesson 7–1)*

54. $p(x) = \frac{1}{2}x^2 + 3x - 1$

55. $p(x) = x^4 - 4x^3 + 2x - 7$

Solve each equation by factoring. *(Lesson 6-3)*

56. $x^2 - 16 = 0$

57. $x^2 - 3x - 10 = 0$

58. $3x^2 + 8x - 3 = 0$

Simplify. *(Lesson 5-6)*

59. $\sqrt{128}$

60. $\sqrt{3x^6y^4}$

61. $\sqrt{20} + 2\sqrt{45} - \sqrt{80}$

Solve each system of equations by using inverse matrices. *(Lesson 4-8)*

62. $x + 2y = 5$
$3x - 3y = -12$

63. $5a + 2b = 4$
$-3a + b = 2$

Find the slope of the line that passes through each pair of points. *(Lesson 2-3)*

64. (2, 1), (5, −3)

65. (0, 4), (7, −2)

66. (5, 3), (2, 3)

Solve each equation. Check your solutions. *(Lesson 1-4)*

67. $|x - 4| = 11$

68. $|2x + 2| = -3$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Evaluate the expression $\frac{x}{x + y}$ for the given values of x and y.** *(To review **evaluating expressions**, see Lesson 1-1.)*

69. $x = 3, y = 2$

70. $x = 4, y = 4$

71. $x = 2, y = 8$

72. $x = 5, y = 10$

12-3 Probability

What You'll Learn

- Find the probability and odds of events.
- Create and use graphs of probability distributions.

Vocabulary

- probability
- success
- failure
- random
- odds
- random variable
- probability distribution
- relative-frequency histogram

What do probability and odds tell you about life's risks?

The risk of getting struck by lightning in any given year is 1 in 750,000. The chances of surviving a lightning strike are 3 in 4. These risks and chances are a way of describing the probability of an event. The **probability** of an event is a ratio that measures the chances of the event occurring.

PROBABILITY AND ODDS Mathematicians often use tossing of coins and rolling of dice to illustrate probability. When you toss a coin, there are only two possible outcomes—heads or tails. A desired outcome is called a **success**. Any other outcome is called a **failure**.

Key Concept Probability of Success and Failure

If an event can succeed in s ways and fail in f ways, then the probabilities of success, $P(S)$, and of failure, $P(F)$, are as follows.

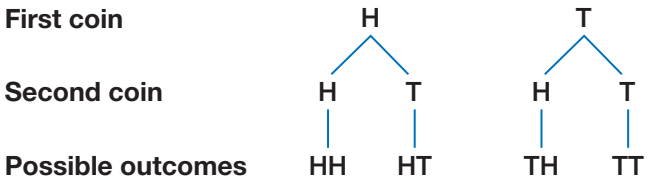
$$P(S) = \frac{s}{s+f} \qquad P(F) = \frac{f}{s+f}$$

The probability of an event occurring is always between 0 and 1, inclusive. The closer the probability of an event is to 1, the more likely the event is to occur. The closer the probability of an event is to 0, the less likely the event is to occur.

Example 1 Probability

When two coins are tossed, what is the probability that both are tails?

You can use a tree diagram to find the sample space.



There are 4 possible outcomes. You can confirm this using the Fundamental Counting Principle. There are 2 possible results for the first coin and 2 for the second coin, so there are $2 \cdot 2$ or 4 possible outcomes. Only one of these outcomes, TT, is a success, so $s = 1$. The other three outcomes are failures, so $f = 3$.

$$P(\text{two tails}) = \frac{s}{s+f} \qquad \text{Probability formula}$$

$$= \frac{1}{1+3} \text{ or } \frac{1}{4} \qquad s = 1, f = 3$$

The probability of tossing two heads is $\frac{1}{4}$. *This probability can also be written as a decimal, 0.25, or as a percent, 25%.*

Study Tip

Reading Math

When P is followed by an event in parentheses, P stands for *probability*. When there are two numbers in parentheses, P stands for *permutations*.

CONTENTS

In more complicated situations, you may need to use permutations and/or combinations to count the outcomes. When all outcomes have an equally likely chance of occurring, we say that the outcomes occur at **random**.

Example 2 Probability with Combinations

Monifa has a collection of 32 CDs—18 R&B and 14 rap. As she is leaving for a trip, she randomly chooses 6 CDs to take with her. What is the probability that she selects 3 R&B and 3 rap?

Step 1 Determine how many 6-CD selections meet the conditions.

$C(18, 3)$ Select 3 R&B CDs. Their order does not matter.
$C(14, 3)$ Select 3 rap CDs.

Step 2 Use the Fundamental Counting Principle to find the number of successes.

$$C(18, 3) \cdot C(14, 3) = \frac{18!}{15!3!} \cdot \frac{14!}{11!3!} \text{ or } 297{,}024$$

Step 3 Find the total number, $s + f$, of possible 6-CD selections.

$$C(32, 6) = \frac{32!}{26!6!} \text{ or } 906{,}192 \qquad s + f = 906{,}192$$

Step 4 Determine the probability.

$$P(\text{3 R\&B CDs and 3 rap CDs}) = \frac{s}{s+f} \qquad \text{Probability formula}$$

$$= \frac{297{,}024}{906{,}192} \qquad \text{Substitute.}$$

$$\approx 0.32777 \qquad \text{Use a calculator.}$$

The probability of selecting 3 R&B CDs and 3 rap CDs is about 0.32777 or 33%.

Another way to measure the chance of an event occurring is with odds. The **odds** that an event will occur can be expressed as the ratio of the number of successes to the number of failures.

Key Concept Odds

The odds that an event will occur can be expressed as the ratio of the number of ways it can succeed to the number of ways it can fail. If an event can succeed in s ways and fail in f ways, then the odds of success and of failure are as follows.

Odds of success $= s:f$ Odds of failure $= f:s$

Example 3 Odds

LIFE EXPECTANCY According to the U.S. National Center for Health Statistics, the chances of a male born in 1990 living to be at least 65 years of age are about 3 in 4. For females, the chances are about 17 in 20.

a. What are the odds of a male living to be at least 65?

Three out of four males will live to be at least 65, so the number of successes (living to 65) is 3. The number of failures is $4 - 3$ or 1.

$$\text{odds of a male living to 65} = s:f \qquad \text{Odds formula}$$

$$= 3:1 \qquad s = 3, f = 1$$

The odds of a male living to at least 65 are 3:1.

b. What are the odds of a female living to be at least 65?

Seventeen out of twenty females will live to be at least 65, so the number of successes in this case is 17. The number of failures is 20 − 17 or 3.

odds of a female living to be 65 $= s{:}f$ Odds formula

$= 17{:}3$ $s = 17, f = 3$

The odds of a female living to at least 65 are 17:3.

PROBABILITY DISTRIBUTIONS

Many experiments, such as rolling a die, have numerical outcomes. A **random variable** is a variable whose value is the numerical outcome of a random event. For example, when rolling a die we can let the random variable D represent the number showing on the die. Then D can equal 1, 2, 3, 4, 5, or 6. A **probability distribution** for a particular random variable is a function that maps the sample space to the probabilities of the outcomes in the sample space. The table below illustrates the probability distribution for rolling a die.

D = Roll	1	2	3	4	5	6
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$P(D = 4) = \frac{1}{6}$

Study Tip

Reading Math

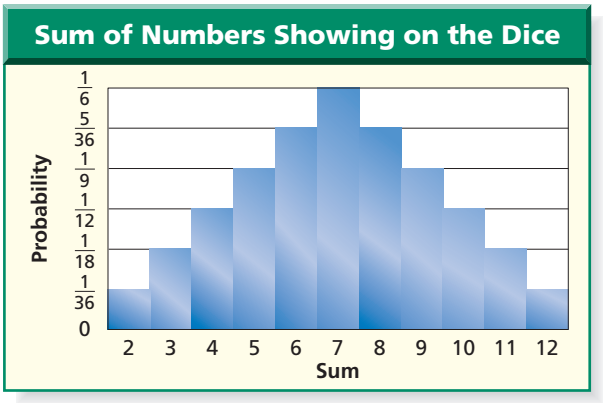
The notation $P(X = n)$ is used with random variables. $P(D = 4) = \frac{1}{6}$ is read *the probability that D equals 4 is one sixth.*

To help visualize a probability distribution, you can use a table of probabilities or a graph, called a **relative-frequency histogram**.

Example 4 Probability Distribution

Suppose two dice are rolled. The table and the relative-frequency histogram show the distribution of the sum of the numbers rolled. *You will be asked to verify some of these probabilities in Exercise 3.*

S = Sum	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$



a. Use the graph to determine which outcome is most likely. What is its probability?

The greatest probability in the graph is $\frac{1}{6}$. The most likely outcome is a sum of 7 and its probability is $\frac{1}{6}$.

b. Use the table to find $P(S = 9)$. What other sum has the same probability?

According to the table, the probability of a sum of 9 is $\frac{1}{9}$. The other outcome with a probability of $\frac{1}{9}$ is 5.

CONTENTS

c. What are the odds of rolling a sum of 7?

Step 1 Identify s and f.

$$P(\text{rolling a } 7) = \frac{1}{6}$$

$$= \frac{s}{s+f} \quad s = 1, f = 5$$

Step 2 Find the odds.

$$\text{Odds} = s{:}f$$

$$= 1{:}5$$

So, the odds of rolling a sum of 7 are 1:5.

Check for Understanding

Concept Check

1. **OPEN ENDED** Describe an event that has a probability of 0 and an event that has a probability of 1.
2. **Write** the probability of an event whose odds are 3:2.
3. **Verify** the probabilities given for sums of 2 and 3 in Example 4.

Guided Practice

Suppose you select 2 letters at random from the word *compute*. Find each probability.

4. $P(2 \text{ vowels})$
5. $P(2 \text{ consonants})$
6. $P(1 \text{ vowel, } 1 \text{ consonant})$

Find the odds of an event occurring, given the probability of the event.

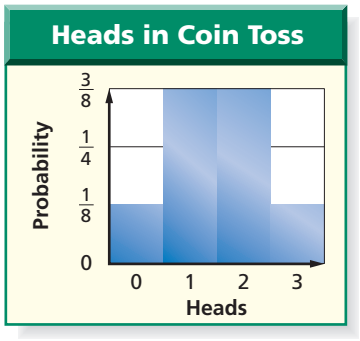
7. $\frac{8}{9}$
8. $\frac{1}{6}$
9. $\frac{2}{9}$

Find the probability of an event occurring, given the odds of the event.

10. 6:5
11. 10:1
12. 2:5

The table and the relative-frequency histogram show the distribution of the number of heads when 3 coins are tossed. Find each probability.

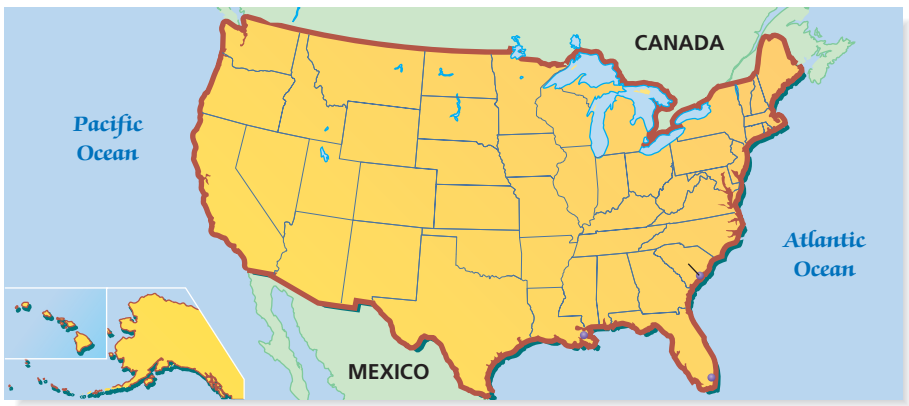
H = Heads	0	1	2	3
Probability	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$



13. $P(H = 0)$
14. $P(H = 2)$

Application

GEOGRAPHY **For Exercises 15–18, find each probability if a state is chosen at random from the 50 states.**



15. $P(\text{next to the Pacific Ocean})$
16. $P(\text{has at least five neighboring states})$
17. $P(\text{borders Mexico})$
18. $P(\text{is surrounded by water})$

Practice and Apply

Homework Help

For Exercises	See Examples
19–33, 54	1, 2
34–53	3
55–60	4

Extra Practice

See page 854.

Ebony has 4 male kittens and 7 female kittens. She picks up 2 kittens to give to a friend. Find the probability of each selection.

19. P(2 male) **20.** P(2 female) **21.** P(1 of each)

Bob is moving and all of his CDs are mixed up in a box. Twelve CDs are rock, eight are jazz, and five are classical. If he reaches in the box and selects them at random, find each probability.

22. P(3 jazz) **23.** P(3 rock)

24. P(1 classical, 2 jazz) **25.** P(2 classical, 1 rock)

26. P(1 jazz, 2 rock) **27.** P(1 classical, 1 jazz, 1 rock)

28. P(2 rock, 2 classical) **29.** P(2 jazz, 1 reggae)

30. LOTTERIES The state of Florida has a lottery in which 6 numbers out of 53 are drawn at random. What is the probability of a given ticket matching all 6 numbers in any order?

ENTRANCE TESTS **For Exercises 31–33, use the table that shows the college majors of the students who took the Medical College Admission Test (MCAT) in April 2000.**
If a student taking the test were randomly selected, find each probability. Express as decimals rounded to the nearest thousandth.

31. P(math or statistics)

32. P(biological sciences)

33. P(physical sciences)

Major	Students
biological sciences	15,819
humanities	963
math or statistics	179
physical sciences	2770
social sciences	2482
specialized health sciences	1431
other	1761

Find the odds of an event occurring, given the probability of the event.

34. $\frac{1}{2}$ **35.** $\frac{3}{8}$ **36.** $\frac{11}{12}$ **37.** $\frac{5}{8}$

38. $\frac{4}{7}$ **39.** $\frac{1}{5}$ **40.** $\frac{4}{11}$ **41.** $\frac{3}{4}$

Find the probability of an event occurring, given the odds of the event.

42. 6:1 **43.** 3:7 **44.** 5:6 **45.** 4:5

46. 9:8 **47.** 1:8 **48.** 7:9 **49.** 3:2

50. GENEOLOGY The odds that an American is of English ancestry are 1:9. What is the probability that an American is of English ancestry?

GENETICS **For Exercises 51 and 52, use the following information.**
Eight out of 100 males and 1 out of 1000 females have some form of color blindness.

51. What are the odds of a male being color-blind?

52. What are the odds of a female being color-blind?

53. EDUCATION Josefina's guidance counselor estimates that the probability she will get a college scholarship is $\frac{4}{5}$. What are the odds that she will *not* earn a scholarship?

More About . . .

Entrance Tests

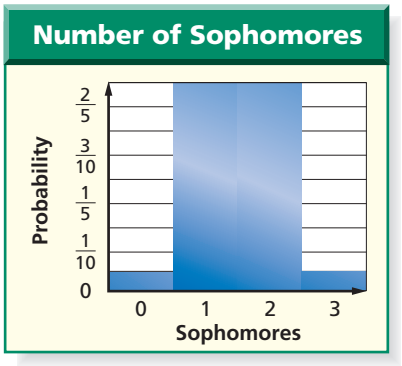
In addition to the MCAT, most medical schools require applicants to have had one year each of biology, physics, and English, and two years of chemistry in college.

CONTENTS

54. **CARD GAMES** The game of euchre is played using only the 9s, 10s, jacks, queens, kings, and aces from a standard deck of cards. Find the probability of being dealt a 5-card euchre hand containing all four suits.

Three students are selected at random from a group of 3 sophomores and 3 juniors. The table and relative-frequency histogram show the distribution of the number of sophomores chosen. Find each probability.

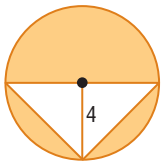
Sophomores	0	1	2	3
Probability	$\frac{1}{20}$	$\frac{9}{20}$	$\frac{9}{20}$	$\frac{1}{20}$



55. P(0 sophomores)
56. P(1 sophomore)
57. P(2 sophomores)
58. P(3 sophomores)
59. P(2 juniors)
60. P(1 junior)

61. **WRITING** Josh types the 5 entries in the bibliography of his term paper in random order, forgetting that they should be in alphabetical order by author. What is the probability that he actually typed them in alphabetical order?

62. **CRITICAL THINKING** Find the probability that a point chosen at random in the figure is in the shaded region. Write your answer in terms of π.



63. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

What do probability and odds tell you about life's risks?

Include the following in your answer:

- the odds of being struck by lightning and surviving the lightning strike, and
- a description of the meaning of *success* and *failure* in this case.

Standardized Test Practice

64. $\frac{6!}{2!} = ?$

(A) 3 (B) 60 (C) 360 (D) 720

65. A jar contains 4 red marbles, 3 green marbles, and 2 blue marbles. If a marble is drawn at random, what is the probability that it is not green?

(A) $\frac{2}{9}$ (B) $\frac{1}{3}$ (C) $\frac{4}{9}$ (D) $\frac{2}{3}$

Extending the Lesson

Theoretical probability is determined using mathematical methods and assumptions about the fairness of coins, dice, and so on. **Experimental probability** is determined by performing experiments and observing the outcomes.

Determine whether each probability is *theoretical* or *experimental*. Then find the probability.

66. Two dice are rolled. What is the probability that the sum will be 12?
67. A baseball player has 126 hits in 410 at-bats this season. What is the probability that he gets a hit in his next at-bat?
68. A bird watcher observes that 5 out of 25 birds in a garden are red. What is the probability that the next bird to fly into the garden will be red?
69. A hand of 2 cards is dealt from a standard deck of cards. What is the probability that both cards are clubs?

Maintain Your Skills

Mixed Review

Determine whether each situation involves a *permutation* or a *combination*. Then find the number of possibilities. *(Lesson 12-2)*

70. arranging 5 different books on a shelf

71. arranging the letters of the word *arrange*

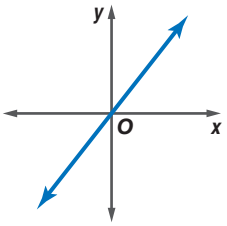
72. picking 3 apples from the last 7 remaining at the grocery store

73. A mail-order computer company offers a choice of 4 amounts of memory, 2 sizes of hard drives, and 2 sizes of monitors. How many different systems are available to a customer? *(Lesson 12-1)*

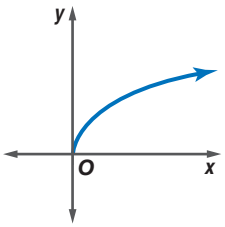
74. How many ways can 4 different gifts be placed into 4 different gift bags if each bag gets exactly 1 gift? *(Lesson 12-1)*

Identify the type of function represented by each graph. *(Lesson 9-5)*

75.



76.



Solve each matrix equation. *(Lesson 4-1)*

77. $[x \quad y] = [y \quad 4]$

78. $\begin{bmatrix} 3y \\ 2x \end{bmatrix} = \begin{bmatrix} x + 8 \\ y - x \end{bmatrix}$

Getting Ready for the Next Lesson

BASIC SKILL **Find each product if $a = \frac{3}{5}$, $b = \frac{2}{7}$, $c = \frac{3}{4}$, and $d = \frac{1}{3}$.**

79. ab **80.** bc **81.** cd **82.** bd **83.** ac

Practice Quiz 1

Lessons 12-1 through 12-3

1. At the Burger Bungalow, you can order your hamburger with or without cheese, with or without onions or pickles, and either rare, medium, or well-done. How many different ways can you order your hamburger? *(Lesson 12-1)*

2. For a particular model of car, a dealer offers 3 sizes of engines, 2 types of stereos, 18 body colors, and 7 upholstery colors. How many different possibilities are available for that model? *(Lesson 12-1)*

3. How many codes consisting of a letter followed by 3 digits can be made if no digit can be used more than once? *(Lesson 12-1)*

Evaluate each expression. *(Lesson 12-2)*

4. $P(12, 3)$ **5.** $C(8, 3)$

Determine whether each situation involves a *permutation* or a *combination*. Then find the number of possibilities. *(Lesson 12-2)*

6. 8 cars in a row parked next to a curb **7.** a hand of 6 cards from a standard deck of cards

Two cards are drawn from a standard deck of cards. Find each probability. *(Lesson 12-3)*

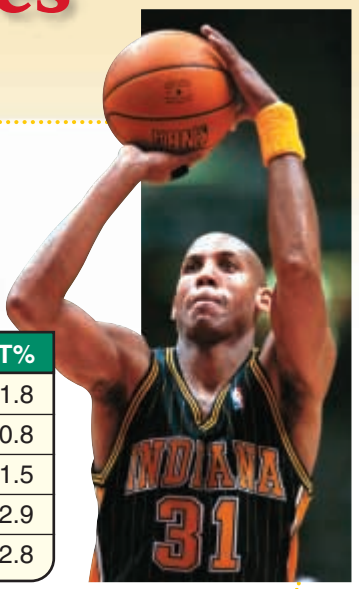
8. P(2 aces) **9.** P(1 heart, 1 club) **10.** P(1 queen, 1 king)

CONTENTS

12-4 Multiplying Probabilities

What You'll Learn

- Find the probability of two independent events.
- Find the probability of two dependent events.



How does probability apply to basketball?

Reggie Miller of the Indiana Pacers is one of the best free-throw shooters in the National Basketball Association. The table shows the five highest season free-throw statistics of his career. For any year, you can determine the probability that Miller will make two free throws in a row based on the probability of his making one free throw.

Season	FT%
1990–91	91.8
1993–94	90.8
1998–99	91.5
1999–00	92.9
2000–01	92.8

Source: *Sporting News*

Vocabulary

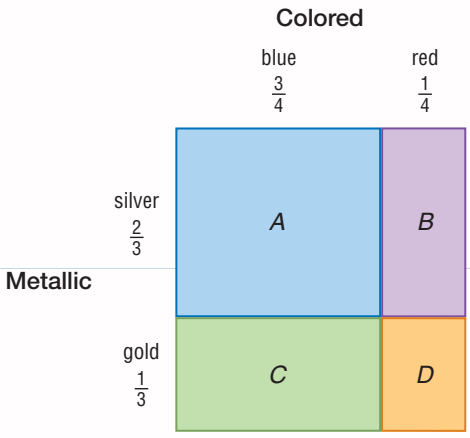
- area diagram

PROBABILITY OF INDEPENDENT EVENTS In a situation with two events like shooting a free throw and then shooting another one, you can find the probability of *both* events occurring if you know the probability of each event occurring. You can use an **area diagram** to model the probability of the two events occurring at the same time.

Algebra Activity

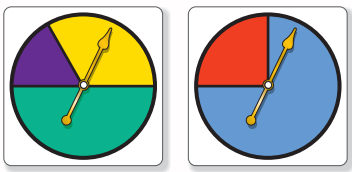
Area Diagrams

Suppose there are 1 red and 3 blue paper clips in one drawer and 1 gold and 2 silver paper clips in another drawer. The area diagram represents the probabilities of choosing one colored paper clip and one metallic paper clip if one of each is chosen at random. For example, rectangle A represents drawing 1 silver clip and 1 blue clip.



Model and Analyze

1. Find the areas of rectangles A, B, C, and D, and explain what each area represents.
2. What is the probability of choosing a red paper clip and a silver paper clip?
3. What are the length and width of the whole square? What is the area? Why does the area need to have this value?
4. Make an area diagram that represents the probability of each outcome if you spin each spinner once. Label the diagram and describe what the area of each rectangle represents.



In Exercise 4 of the activity, spinning one spinner has no effect on the second spinner. These events are independent.

Key Concept — **Probability of Two Independent Events**

If two events, A and B, are independent, then the probability of both events occurring is $P(A \text{ and } B) = P(A) \cdot P(B)$.

This formula can be applied to any number of independent events.

Example 1 Two Independent Events

At a picnic, Julio reaches into an ice-filled cooler containing 8 regular soft drinks and 5 diet soft drinks. He removes a can, then decides he is not really thirsty, and puts it back. What is the probability that Julio and the next person to reach into the cooler both randomly select a regular soft drink?

Explore These events are independent since Julio replaced the can that he removed. The outcome of the second person's selection is not affected by Julio's selection.

Plan Since there are 13 cans, the probability of each person's getting a regular soft drink is $\frac{8}{13}$.

Solve

$$P(\text{both regular}) = P(\text{regular}) \cdot P(\text{regular})$$

Probability of independent events

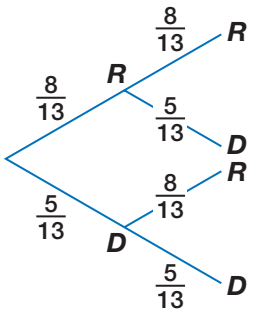
$$= \frac{8}{13} \cdot \frac{8}{13} \text{ or } \frac{64}{169}$$

Substitute and multiply.

The probability that both people select a regular soft drink is $\frac{64}{169}$ or about 0.38.

Examine You can verify this result by making a tree diagram that includes probabilities. Let R stand for regular and D stand for diet.

$$P(R, R) = \frac{8}{13} \cdot \frac{8}{13}$$



Study Tip

Alternative Method

You could use the Fundamental Counting Principle to find the number of successes and the number of total outcomes.

both regular $= 8 \cdot 8$ or 64

total outcomes $= 13 \cdot 13$ or 169

So, $P(\text{both reg.}) = \frac{64}{169}$.

The formula for the probability of independent events can be extended to any number of independent events.

Example 2 Three Independent Events

In a board game, three dice are rolled to determine the number of moves for the players. What is the probability that the first die shows a 6, the second die shows a 6, and the third die does not?

Let A be the event that the first die shows a 6. → $P(A) = \frac{1}{6}$

Let B be the event that the second die shows a 6. → $P(B) = \frac{1}{6}$

Let C be the event that the third die does *not* show a 6. → $P(C) = \frac{5}{6}$

CONTENTS

$P(A, B, \text{and } C) = P(A) \cdot P(B) \cdot P(C)$ Probability of independent events

$= \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{5}{6}$ or $\frac{5}{216}$ Substitute and multiply.

The probability that the first die shows a 6 and the second die does not is $\frac{5}{36}$.

PROBABILITY OF DEPENDENT EVENTS

In Example 1, what is the probability that both people select a regular soft drink if Julio does not put his back in the cooler? In this case, the two events are dependent because the outcome of the first event affects the outcome of the second event.

Study Tip

Conditional Probability

The event of getting a regular soft drink the second time *given* that Julio got a regular soft drink the first time is called a *conditional probability*.

First selection	**Second selection**	
$P(\text{regular}) = \frac{8}{13}$	$P(\text{regular}) = \frac{7}{12}$	Notice that when Julio removes his can, there is not only one fewer regular soft drink but also one fewer drink in the cooler.

$P(\text{both regular}) = P(\text{regular}) \cdot P(\text{regular following regular})$

$= \frac{8}{13} \cdot \frac{7}{12}$ or $\frac{14}{39}$ Substitute and multiply.

The probability that both people select a regular soft drink is $\frac{14}{39}$ or about 0.36.

Key Concept — Probability of Two Dependent Events

If two events, A and B, are dependent, then the probability of both events occurring is $P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$.

This formula can be extended to any number of dependent events.

Example 3 Two Dependent Events

The host of a game show is drawing chips from a bag to determine the prizes for which contestants will play. Of the 10 chips in the bag, 6 show *television*, 3 show *vacation*, and 1 shows *car*. If the host draws the chips at random and does not replace them, find each probability.

Because the first chip is not replaced, the events are dependent. Let T represent a television, V a vacation, and C a car.

a. a vacation, then a car

$P(V, \text{then } C) = P(V) \cdot P(C \text{ following } V)$ Dependent events

$= \frac{3}{10} \cdot \frac{1}{9}$ or $\frac{1}{30}$ After the first chip is drawn, there are 9 left.

The probability of a vacation and then a car is $\frac{1}{30}$ or about 0.03.

b. two televisions

$P(T, \text{then } T) = P(T) \cdot P(T \text{ following } T)$ Dependent events

$= \frac{6}{10} \cdot \frac{5}{9}$ or $\frac{1}{3}$ If the first chip shows television, then 5 of the remaining 9 show television.

The probability of the host drawing two televisions is $\frac{1}{3}$.

Example 4 Three Dependent Events

Three cards are drawn from a standard deck of cards without replacement. Find the probability of drawing a diamond, a club, and another diamond in that order.

Since the cards are not replaced, the events are dependent. Let D represent a diamond and C a club.

$$P(D, C, D) = P(D) \cdot P(C \text{ following } D) \cdot P(D \text{ following } D \text{ and } C)$$
$$= \frac{13}{52} \cdot \frac{13}{51} \cdot \frac{12}{50} \text{ or } \frac{13}{850}$$

If the first two cards are a diamond and a club, then 12 of the remaining cards are diamonds.

The probability is $\frac{13}{850}$ or about 0.015.

Check for Understanding

Concept Check

1. **OPEN ENDED** Describe two real-life events that are dependent.
2. **Write** a formula for $P(A, B, C, \text{and } D)$ if A, B, C, and D are independent.
3. **FIND THE ERROR** Mario and Tabitha are calculating the probability of getting a 4 and then a 2 if they roll a die twice.

Mario

$P(4, \text{then } 2) = \frac{1}{6} \cdot \frac{1}{6}$

$= \frac{1}{36}$

Tabitha

$P(4, \text{then } 2) = \frac{1}{6} \cdot \frac{1}{5}$

$= \frac{1}{30}$

Who is correct? Explain your reasoning.

Guided Practice

A die is rolled twice. Find each probability.

4. $P(5, \text{then } 1)$
5. $P(\text{two even numbers})$

Two cards are drawn from a standard deck of cards. Find each probability if no replacement occurs.

6. $P(\text{two hearts})$
7. $P(\text{ace, then king})$

There are 8 action, 3 romantic comedy, and 5 children's DVDs on a shelf. Suppose two DVDs are selected at random from the shelf. Find each probability.

8. $P(2 \text{ action DVDs})$, if no replacement occurs
9. $P(2 \text{ action DVDs})$, if replacement occurs
10. $P(\text{a romantic comedy DVD, then a children's DVD})$, if no replacement occurs

Determine whether the events are *independent* or *dependent*. Then find the probability.

11. Yana has 7 blue pens, 3 black pens, and 2 red pens in his desk drawer. If he selects three pens at random with no replacement, what is the probability that he will first select a blue pen, then a black pen, and then another blue pen?
12. A black die and a white die are rolled. What is the probability that a 3 shows on the black die and a 5 shows on the white die?

Application

13. ELECTIONS Tami, Sonia, Malik, and Roger are the four candidates for student council president. If their names are placed in random order on the ballot, what is the probability that Malik's name will be first on the ballot followed by Sonia's name second?

Practice and Apply

Homework Help

For Exercises	See Examples
14–19, 36–39, 44–46	1, 2
20–29	1, 3
30–35	1–4
40–43	3

Extra Practice

See page 855.

A die is rolled twice. Find each probability.

14. P(2, then 3)

15. P(no 6s)

16. P(two 4s)

17. P(1, then any number)

18. P(two of the same number)

19. P(two different numbers)

The tiles *A, B, G, I, M, R,* and *S* of a word game are placed face down in the lid of the game. If two tiles are chosen at random, find each probability.

20. P(R, then S), if no replacement occurs

21. P(A, then M), if replacement occurs

22. P(2 consonants), if replacement occurs

23. P(2 consonants), if no replacement occurs

24. P(B, then D), if replacement occurs

25. P(selecting the same letter twice), if no replacement occurs

Ashley takes her 3-year-old brother Alex into an antique shop. There are 4 statues, 3 picture frames, and 3 vases on a shelf. Alex accidentally knocks 2 items off the shelf and breaks them. Find each probability.

26. P(breaking 2 vases)

27. P(breaking 2 statues)

28. P(breaking a picture frame, then a vase)

29. P(breaking a statue, then a picture frame)

Determine whether the events are *independent* or *dependent*. Then find the probability.

30. There are 3 miniature chocolate bars and 5 peanut butter cups in a candy dish. Judie chooses 2 of them at random. What is the probability that she chooses 2 miniature chocolate bars?

31. A bowl contains 4 peaches and 5 apricots. Maxine randomly selects one, puts it back, and then randomly selects another. What is the probability that both selections were apricots?

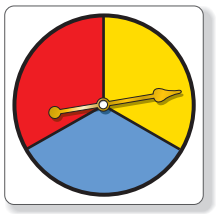
32. A bag contains 7 red, 4 blue, and 6 yellow marbles. If 3 marbles are selected in succession, what is the probability of selecting blue, then yellow, then red, if replacement occurs each time?

33. Joe's wallet contains three $1 bills, four $5 bills, and two $10 bills. If he selects three bills in succession, find the probability of selecting a $10 bill, then a $5 bill, and then a $1 bill if the bills are not replaced.

34. What is the probability of getting heads each time if a coin is tossed 5 times?

35. When Diego plays his favorite video game, the odds are 3 to 4 that he will reach the highest level of the game. What is the probability that he will reach the highest level each of the next four times he plays?

For Exercises 36–39, suppose you spin the spinner twice.



36. Sketch a tree diagram showing all of the possibilities. Use it to find the probability of spinning red and then blue.

37. Sketch an area diagram of the outcomes. Shade the region on your area diagram corresponding to getting the same color twice.

38. What is the probability that you get the same color on both spins?

39. If you spin the same color twice, what is the probability that the color is red?

Find each probability if 13 cards are drawn from a standard deck of cards and no replacement occurs.

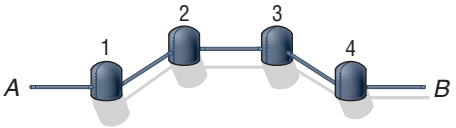
40. *P*(all clubs)

41. *P*(all black cards)

42. *P*(all one suit)

43. *P*(no aces)

44. **UTILITIES** A city water system includes a sequence of 4 pumps as shown below. Water enters the system at point A, is pumped through the system by pumps at locations 1, 2, 3, and 4, and exits the system at point B.



If the probability of failure for any one pump is $\frac{1}{100}$, what is the probability that water will flow all the way through the system from A to B?

45. **SPELLING** Suppose a contestant in a spelling bee has a 93% chance of spelling any given word correctly. What is the probability that he or she spells the first five words in a bee correctly and then misspells the sixth word?

46. **LITERATURE** The following quote is from *The Mirror Crack'd,* which was written by Agatha Christie in 1962.

> "I think you're begging the question," said Haydock, "and I can see looming ahead one of those terrible exercises in probability where six men have white hats and six men have black hats and you have to work it out by mathematics how likely it is that the hats will get mixed up and in what proportion. If you start thinking about things like that, you would go round the bend. Let me assure you of that!"

If the twelve hats are all mixed up and each man randomly chooses a hat, what is the probability that the first three men get their own hats? Assume that no replacement occurs.

For Exercises 47–49, use the following information.
You have a bag containing 10 marbles. In this problem, a *cycle* means that you draw a marble, record its color, and put it back.

47. You go through the cycle 10 times. If you do not record any black marbles, can you conclude that there are no black marbles in the bag?

48. Can you conclude that there are none if you repeat the cycle 50 times?

49. How many times do you have to repeat the cycle to be certain that there are no black marbles in the bag? Explain your reasoning.

50. **CRITICAL THINKING** If one bulb in a string of holiday lights fails to work, the whole string will not light. If each bulb in a set has a 99.5% chance of working, what is the maximum number of lights that can be strung together with at least a 90% chance of the whole string lighting?

More About . . .

Spelling

The National Spelling Bee has been held every year since 1925, except for 1943-1945. Of the first 76 champions, 42 were girls and 34 were boys.

Source: www.spellingbee.com

CONTENTS

51. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

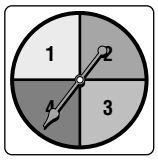
How does probability apply to basketball?

Include the following in your answer:

- an explanation of how a value such as one of those in the table at the beginning of the lesson could be used to find the chances of Reggie Miller making 0, 1, or 2 of 2 successive free throws, assuming the 2 free throws are independent, and
- a possible psychological reason why 2 free throws on the same trip to the foul line might not be independent.

Standardized Test Practice

52. The spinner is spun four times. What is the probability that the spinner lands on 2 each time?



(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{8}$ (D) $\frac{1}{16}$

53. A coin is tossed and a die is rolled. What is the probability of a head and a 3?

(A) $\frac{1}{4}$ (B) $\frac{1}{8}$ (C) $\frac{1}{12}$ (D) $\frac{1}{24}$

Maintain Your Skills

Mixed Review

A gumball machine contains 7 red, 8 orange, 9 purple, 7 white, and 5 yellow gumballs. Tyson buys 3 gumballs. Find each probability, assuming that the machine dispenses the gumballs at random. *(Lesson 12-3)*

54. P(3 red)

55. P(2 white, 1 purple)

56. P(1 purple, 1 orange, 1 yellow)

57. PHOTOGRAPHY A photographer is taking a picture of a bride and groom together with 6 attendants. How many ways can he arrange the 8 people in a row if the bride and groom stand in the middle? *(Lesson 12-2)*

Solve each equation. Check your solutions. *(Lesson 10-3)*

58. $\log_5 5 + \log_5 x = \log_5 30$

59. $\log_{16} c - 2\log_{16} 3 = \log_{16} 4$

Given a polynomial and one of its factors, find the remaining factors of the polynomial. Some factors may not be binomials. *(Lesson 7-4)*

60. $x^3 - x^2 - 10x + 6; x + 3$

61. $x^3 - 7x^2 + 12x; x - 3$

Graph each inequality. *(Lesson 6-7)*

62. $y \leq x^2 + x - 2$

63. $y < x^2 - 4$

64. $y > x^2 - 3x$

Simplify. *(Lesson 5-5)*

65. $\sqrt{(153)^2}$

66. $\sqrt[3]{-729}$

67. $\sqrt[16]{b^{16}}$

68. $\sqrt{25a^8b^6}$

Solve each system of equations. *(Lesson 3-2)*

69. $z = 4y - 2$
$z = -y + 3$

70. $j - k = 4$
$2j + k = 35$

71. $3x + 1 = -y - 1$
$2y = -4x$

Getting Ready for the Next Lesson

BASIC SKILL **Find each sum if $a = \frac{1}{2}$, $b = \frac{1}{6}$, $c = \frac{2}{3}$, and $d = \frac{3}{4}$.**

72. $a + b$

73. $b + c$

74. $a + d$

75. $b + d$

76. $c + a$

77. $c + d$

12-5 Adding Probabilities

What You'll Learn

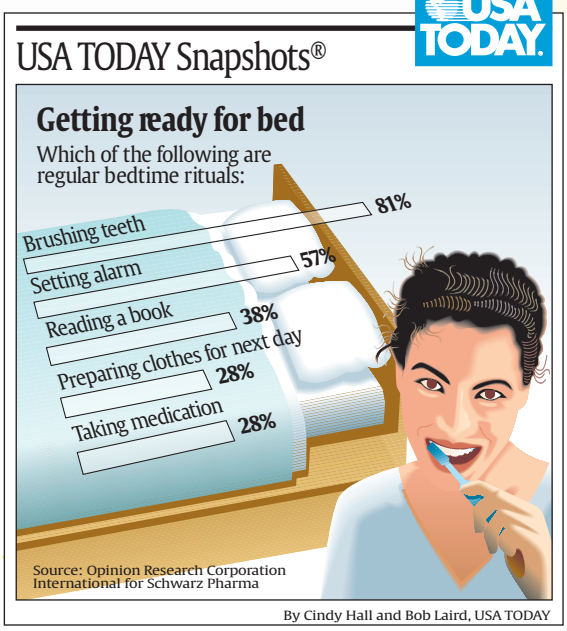
- Find the probability of mutually exclusive events.
- Find the probability of inclusive events.

Vocabulary

- simple event
- compound event
- mutually exclusive events
- inclusive events

How does probability apply to your personal habits?

The graph shows the results of a survey about bedtime rituals. Determining the probability that a randomly selected person reads a book or brushes his or her teeth before going to bed requires adding probabilities.



MUTUALLY EXCLUSIVE EVENTS When you roll a die, an event such as rolling a 1 is called a **simple event** because it consists of only one event. An event that consists of two or more simple events is called a **compound event**. For example, the event of rolling an odd number or a number greater than 5 is a compound event because it consists of the simple events rolling a 1, rolling a 3, rolling a 5, or rolling a 6.

When there are two events, it is important to understand how they are related before finding the probability of one or the other event occurring. Suppose you draw a card from a standard deck of cards. What is the probability of drawing a 2 or an ace? Since a card cannot be both a 2 *and* an ace, these are called **mutually exclusive events**. That is, the two events cannot occur at the same time. The probability of drawing a 2 or an ace is found by adding their individual probabilities.

$$\begin{aligned} P(\text{2 or ace}) &= P(2) + P(\text{ace}) && \text{Add probabilities.} \\ &= \frac{4}{52} + \frac{4}{52} && \text{There are 4 twos and 4 aces in a deck.} \\ &= \frac{8}{52} \text{ or } \frac{2}{13} && \text{Simplify.} \end{aligned}$$

The probability of drawing a 2 or an ace is $\frac{2}{13}$.

Key Concept — *Probability of Mutually Exclusive Events*

- **Words** If two events, A and B, are mutually exclusive, then the probability that A or B occurs is the sum of their probabilities.
- **Symbols** $P(A \text{ or } B) = P(A) + P(B)$

This formula can be extended to any number of mutually exclusive events.

CONTENTS

Example 1 Two Mutually Exclusive Events

Keisha has a stack of 8 baseball cards, 5 basketball cards, and 6 soccer cards. If she selects a card at random from the stack, what is the probability that it is a baseball or a soccer card?

These are mutually exclusive events, since the card cannot be both a baseball card *and* a soccer card. Note that there is a total of 19 cards.

$$P(\text{baseball or soccer}) = P(\text{baseball}) + P(\text{soccer}) \quad \text{Mutually exclusive events}$$

$$= \frac{8}{19} + \frac{6}{19} \text{ or } \frac{14}{19} \quad \text{Substitute and add.}$$

The probability that Keisha selects a baseball or a soccer card is $\frac{14}{19}$.

Example 2 Three Mutually Exclusive Events

There are 7 girls and 6 boys on the junior class homecoming committee. A subcommittee of 4 people is being chosen at random to decide the theme for the class float. What is the probability that the subcommittee will have at least 2 girls?

At least 2 girls means that the subcommittee may have 2, 3, or 4 girls. It is not possible to select a group of 2 girls, a group of 3 girls, and a group of 4 girls all in the same 4-member subcommittee, so the events are mutually exclusive. Add the probabilities of each type of committee.

$$P(\text{at least 2 girls}) = P(2 \text{ girls}) + P(3 \text{ girls}) + P(4 \text{ girls})$$

$$= \frac{C(7, 2) \cdot C(6, 2)}{C(13, 4)} + \frac{C(7, 3) \cdot C(6, 1)}{C(13, 4)} + \frac{C(7, 4) \cdot C(6, 0)}{C(13, 4)}$$

(2 girls, 2 boys; 3 girls, 1 boy; 4 girls, 0 boys)

$$= \frac{315}{715} + \frac{210}{715} + \frac{35}{715} \text{ or } \frac{112}{143} \quad \text{Simplify.}$$

The probability of at least 2 girls on the subcommittee is $\frac{112}{143}$ or about 0.78.

Study Tip

Choosing a Committee

$C(13, 4)$ refers to choosing 4 subcommittee members from 13 committee members. Since order does not matter, the number of combinations is found.

INCLUSIVE EVENTS What is the probability of drawing a queen or a diamond from a standard deck of cards? Since it is possible to draw a card that is both a queen and a diamond, these events are *not* mutually exclusive. These are called **inclusive events**.

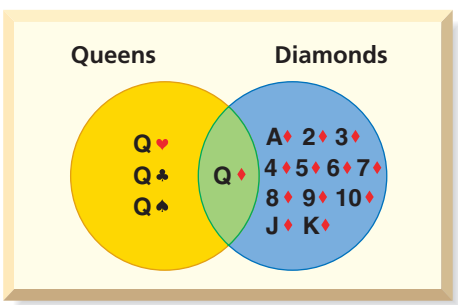
P(queen)	P(diamond)	P(diamond, queen)
$\frac{4}{52}$	$\frac{13}{52}$	$\frac{1}{52}$
1 queen in each suit	diamonds	queen of diamonds

Study Tip

Common Misconception

In mathematics, unlike everyday language, the expression *A* or *B* allows the possibility of both *A* and *B* occurring.

In the first two fractions above, the probability of drawing the queen of diamonds is counted twice, once for a queen and once for a diamond. To find the correct probability, you must subtract P(queen of diamonds) from the sum of the first two probabilities.



$$P(\text{queen or diamond}) = P(\text{queen}) + P(\text{diamond}) - P(\text{queen of diamonds})$$
$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} \text{ or } \frac{4}{13}$$

The probability of drawing a queen or a diamond is $\frac{4}{13}$.

Key Concept — *Probability of Inclusive Events*

- **Words** If two events, A and B, are inclusive, then the probability that A or B occurs is the sum of their probabilities decreased by the probability of both occurring.
- **Symbols** $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Example 3 *Inclusive Events*

EDUCATION **The enrollment at Southburg High School is 1400. Suppose 550 students take French, 700 take algebra, and 400 take both French and algebra. What is the probability that a student selected at random takes French or algebra?**

Since some students take both French and algebra, the events are inclusive.

$$P(\text{French}) = \frac{550}{1400} \qquad P(\text{algebra}) = \frac{700}{1400} \qquad P(\text{French and algebra}) = \frac{400}{1400}$$

$$P(\text{French or algebra}) = P(\text{French}) + P(\text{algebra}) - P(\text{French and algebra})$$
$$= \frac{550}{1400} + \frac{700}{1400} - \frac{400}{1400} \text{ or } \frac{17}{28} \quad \text{Substitute and simplify.}$$

The probability that a student selected at random takes French or algebra is $\frac{17}{28}$.

Check for Understanding

Concept Check

1. **OPEN ENDED** Describe two mutually exclusive events and two inclusive events.
2. **Draw** a Venn diagram to illustrate Example 3.
3. **FIND THE ERROR** Refer to the comic below.

The Born Loser®

Why is the weather forecaster's prediction incorrect?

Guided Practice

A die is rolled. Find each probability.

4. $P(1 \text{ or } 6)$
5. $P(\text{at least } 5)$
6. $P(\text{less than } 3)$
7. $P(\text{prime})$
8. $P(\text{even or prime})$
9. $P(\text{multiple of 2 or 3})$

A card is drawn from a standard deck of cards. Determine whether the events are *mutually exclusive* or *inclusive*. Then find the probability.

10. P(6 or king)

11. P(queen or spade)

Application

12. SCHOOL There are 8 girls and 8 boys on the student senate. Three of the students are seniors. What is the probability that a person selected from the student senate is not a senior?

Practice and Apply

Homework Help

For Exercises	See Examples
13–22, 33–42	1, 2
23–26	1–3
27–32, 43–46	3

Extra Practice

See page 855.

Lisa has 9 rings in her jewelry box. Five are gold and 4 are silver. If she randomly selects 3 rings to wear to a party, find each probability.

13. P(2 silver or 2 gold)

14. P(all gold or all silver)

15. P(at least 2 gold)

16. P(at least 1 silver)

Seven girls and six boys walk into a video store at the same time. There are five salespeople available to help them. Find the probability that the salespeople will first help the given numbers of girls and boys.

17. P(4 girls or 4 boys)

18. P(3 girls or 3 boys)

19. P(all girls or all boys)

20. P(at least 3 girls)

21. P(at least 4 girls or at least 4 boys)

22. P(at least 2 boys)

For Exercises 23–26, determine whether the events are *mutually exclusive* or *inclusive*. Then find the probability.

23. There are 3 literature books, 4 algebra books, and 2 biology books on a shelf. If a book is randomly selected, what is the probability of selecting a literature book or an algebra book?

24. A die is rolled. What is the probability of rolling a 5 or a number greater than 3?

25. In the Math Club, 7 of the 20 girls are seniors, and 4 of the 14 boys are seniors. What is the probability of randomly selecting a boy or a senior to represent the Math Club at a statewide math contest?

26. A card is drawn from a standard deck of cards. What is the probability of drawing an ace or a face card? (*Hint:* A face card is a jack, queen, or king.)

27. One tile with each letter of the alphabet is placed in a bag, and one is drawn at random. What is the probability of selecting a vowel or a letter from the word *equation*?

28. Each of the numbers from 1 to 30 is written on a card and placed in a bag. If one card is drawn at random, what is the probability that the number is a multiple of 2 or a multiple of 3?

Two cards are drawn from a standard deck of cards. Find each probability.

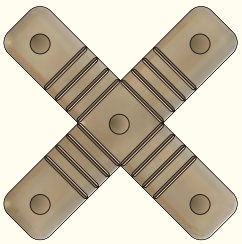
29. P(both kings or both black)

30. P(both kings or both face cards)

31. P(both face cards or both red)

32. P(both either red or a king)

More About . . .



World Cultures

Totolospi is a Hopi game of chance. The players use cane dice, which have both a flat side and a round side, and a counting board inscribed in stone.

WORLD CULTURES **For Exercises 33–36, refer to the information at the left.** When tossing 3 cane dice, if three round sides land up, the player advances 2 lines. If three flat sides land up, the player advances 1 line. If a combination is thrown, the player loses a turn. Find each probability.

33. P(advancing 2 lines)

34. P(advancing 1 line)

35. P(advancing at least 1 line)

36. P(losing a turn)

For Exercises 37–42, use the following information.
Each of the numbers 1 through 30 is written on a table tennis ball and placed in a wire cage. Each of the numbers 20 through 45 is written on a table tennis ball and placed in a different wire cage. One ball is chosen at random from each spinning cage. Find each probability.

37. P(each is a 25)

38. P(neither is a 20)

39. P(exactly one is a 30)

40. P(exactly one is a 40)

41. P(the numbers are equal)

42. P(the sum is 30)

More About . . .

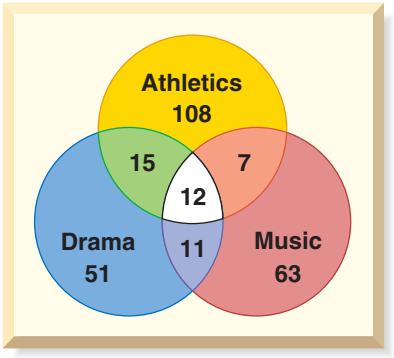
Recycling

The United States recycles 28% of its waste.

Source: The U.S. Environmental Protection Agency

43. RECYCLING In one community, 300 people were surveyed to see if they would participate in a curbside recycling program. Of those surveyed, 134 said they would recycle aluminum cans, and 108 said they would recycle glass. Of those, 62 said they would recycle both. What is the probability that a randomly selected member of the community would recycle aluminum or glass?

SCHOOL **For Exercises 44–46, use the Venn diagram that shows the number of participants in extracurricular activities for a junior class of 324 students.**
Determine each probability if a student is selected at random from the class.



44. P(drama or music)

45. P(drama or athletics)

46. P(athletics and drama, or music and athletics)

47. CRITICAL THINKING Consider the following probability equation.

$$P(A \text{ and } B) = P(A) + P(B) - P(A \text{ or } B)$$

A textbook gives this equation for events A and B that are mutually exclusive or inclusive. Is this correct? Explain.

48. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How does probability apply to your personal habits?

Include the following in your answer:

- an explanation of whether the events listed in the graphic are mutually exclusive or inclusive, and
- an explanation of how to determine the probability that a randomly selected person reads a book or brushes his or her teeth before going to bed if in a survey of 2000 people, 600 said that they do both.

Standardized Test Practice

A B C D

49. In a jar of red and white gumballs, the ratio of white gumballs to red gumballs is 5:4. If the jar contains a total of 180 gumballs, how many of them are red?

(A) 45 (B) 64 (C) 80 (D) 100

50. $\langle x \rangle = \frac{1}{2}x$ if x is composite. $\langle x \rangle = 2x$ if x is prime. What is the value of $\langle 7 \rangle + \langle 18 \rangle$?

(A) 23 (B) 46 (C) 50 (D) 64

CONTENTS

Maintain Your Skills

Mixed Review

A die is rolled three times. Find each probability. *(Lesson 12-4)*

51. P(1, then 2, then 3)

52. P(no 4s)

53. P(three 1s)

54. P(three even numbers)

Find the odds of an event occurring, given the probability of the event. *(Lesson 12-3)*

55. $\frac{4}{5}$ **56.** $\frac{1}{9}$ **57.** $\frac{2}{7}$ **58.** $\frac{5}{8}$

Find the sum of each series. *(Lessons 11-2 and 11-4)*

59. $2 + 4 + 8 + \cdots + 128$

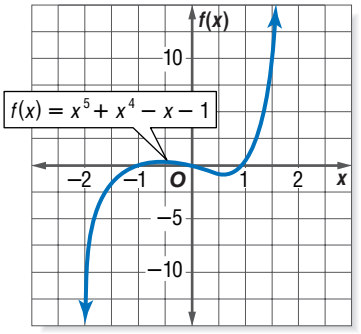
60. $\sum_{n=1}^{3} (5n - 2)$

Find the exact solution(s) of each system of equations. *(Lesson 8-7)*

61. $y = -10$
$y^2 = x^2 + 36$

62. $x^2 = 144$
$x^2 + y^2 = 169$

63. Use the graph of the polynomial function at the right to determine at least one binomial factor of the polynomial. Then find all factors of the polynomial. *(Lesson 7-4)*



Find the maxima and minima of each function. Round to the nearest hundredth. *(Lesson 6-2)*

64. $f(x) = x^3 + 2x^2 - 5$

65. $f(x) = x^3 + 3x^2 + 2x + 1$

Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region. *(Lesson 3-4)*

66. $y \geq x - 2$
$x \geq 0$
$y \leq 2 - x$
$f(x, y) = 3x + y$

67. $y \geq 2x - 3$
$1 \leq x \leq 3$
$y \leq x + 2$
$f(x, y) = x + 4y$

SPEED SKATING **For Exercises 68 and 69, use the following information.**
In the 1988 Winter Olympics, Bonnie Blair set a world record for women's speed skating by skating approximately 12.79 meters per second in the 500-meter race. *(Lesson 2-6)*

68. Suppose she could maintain that speed. Write an equation that represents how far she could travel in t seconds.

69. What type of equation is the one in Exercise 68?

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find the mean, median, mode, and range for each set of data. Round to the nearest hundredth, if necessary.**
*(To review **mean**, **median**, **mode**, and **range**, see pages 822 and 823.)*

70. 298, 256, 399, 388, 276

71. 3, 75, 58, 7, 34

72. 4.8, 5.7, 2.1, 2.1, 4.8, 2.1

73. 80, 50, 65, 55, 70, 65, 75, 50

74. 61, 89, 93, 102, 45, 89

75. 13.3, 15.4, 12.5, 10.7

12-6 Statistical Measures

What You'll Learn

- Use measures of central tendency to represent a set of data.
- Find measures of variation for a set of data.

What statistics should a teacher tell the class after a test?

On Mr. Dent's most recent Algebra 2 test, his students earned the following scores.

72	70	77	76	90	68	81	86	34	94
71	84	89	67	19	85	75	66	80	94

When his students ask how they did on the test, which measure of central tendency should Mr. Dent use to describe the scores?

Vocabulary

- measure of central tendency
- measure of variation
- dispersion
- variance
- standard deviation

MEASURES OF CENTRAL TENDENCY

Sometimes it is convenient to have one number that describes a set of data. This number is called a **measure of central tendency**, because it represents the center or middle of the data. The most commonly used measures of central tendency are the *mean*, *median*, and *mode*.

When deciding which measure of central tendency to use to represent a set of data, look closely at the data itself.

Concept Summary Measures of Tendency

Use	When . . .
mean	the data are spread out, and you want an average of the values.
median	the data contain outliers.
mode	the data are tightly clustered around one or two values.

Study Tip

Look Back

To review **outliers**, see Lesson 2-5.

Example 1 Choose a Measure of Central Tendency

SWEEPSTAKES **A sweepstakes offers a first prize of $10,000, two second prizes of $100, and one hundred third prizes of $10.**

a. Which measure of central tendency best represents the available prizes?

Since 100 of the 103 prizes are $10, the mode ($10) best represents the available prizes. Notice that in this case the median is the same as the mode.

b. Which measure of central tendency would the organizers of the sweepstakes be most likely to use in their advertising?

The organizers would be most likely to use the mean (about $109) to make people think they had a better chance of winning more money.

MEASURES OF VARIATION

Measures of variation or **dispersion** measure how spread out or scattered a set of data is. The simplest measure of variation to calculate is the *range*, the difference between the greatest and the least values in a set of data. Variance and standard deviation are measures of variation that indicate how much the data values differ from the mean.

Study Tip

Reading Math
The symbol σ is the lower case Greek letter *sigma*. $\bar{x}$ is read *x bar*.

To find the **variance** σ^2 of a set of data, follow these steps.

1. Find the mean, $\bar{x}$.
2. Find the difference between each value in the set of data and the mean.
3. Square each difference.
4. Find the mean of the squares.

The **standard deviation** σ is the square root of the variance.

Key Concept — **Standard Deviation**

If a set of data consists of the n values $x_1, x_2, \ldots, x_n$ and has mean $\bar{x}$, then the standard deviation σ is given by the following formula.

$$\sigma = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n}}$$

Example 2 Standard Deviation

STATES **The table shows the populations in millions of 11 eastern states as of the 2000 Census. Find the variance and standard deviation of the data to the nearest tenth.**

State	Population	State	Population	State	Population
NY	19.0	MD	5.3	RI	1.0
PA	12.3	CT	3.4	DE	0.8
NJ	8.4	ME	1.3	VT	0.6
MA	6.3	NH	1.2	—	—

Source: U.S. Census Bureau

Step 1 Find the mean. Add the data and divide by the number of items.

$$\bar{x} = \frac{19.0 + 12.3 + 8.4 + 6.3 + 5.3 + 3.4 + 1.3 + 1.2 + 1.0 + 0.8 + 0.6}{11}$$

$\approx 5.4\overline{18}$ The mean is about 5.4 people.

Step 2 Find the variance.

$$\sigma^2 = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n}$$ Variance formula

$$\approx \frac{(19.0 - 5.4)^2 + (12.3 - 5.4)^2 + \cdots + (0.8 - 5.4)^2 + (0.6 - 5.4)^2}{11}$$

$\approx \frac{344.4}{11}$ Simplify.

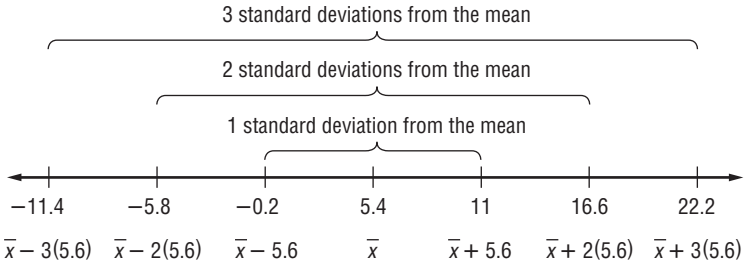
$\approx 31.3\overline{09}$ The variance is about 31.3 people.

Step 3 Find the standard deviation.

$\sigma^2 \approx 31.3$ Take the square root of each side.

$\sigma \approx 5.594640292$ The standard deviation is about 5.6 people.

Most of the members of a set of data are within 1 standard deviation of the mean. The populations of the states in Example 2 can be broken down as shown below.



Looking at the original data, you can see that most of the states' populations were between 2.4 million and 20.2 million. That is, the majority of members of the data set were within 1 standard deviation of the mean.

You can use a TI-83 Plus graphing calculator to find statistics for the data in Example 2.

Graphing Calculator Investigation

One-Variable Statistics

The TI-83 Plus can compute a set of one-variable statistics from a list of data. These statistics include the mean, variance, and standard deviation. Enter the data into L1.

KEYSTROKES: STAT ENTER 19.0 ENTER 12.3 ENTER ...

Then use STAT ▶ 1 ENTER to show the statistics. The mean $\bar{x}$ is about 5.4, the sum of the values $\sum x$ is 59.6, the standard deviation σx is about 5.6, and there are $n = 11$ data items. If you scroll down, you will see the least value (minX = .6), the three quartiles (1, 3.4, and 8.4), and the greatest value (maxX = 19).

Think and Discuss

1. Find the variance of the data set.
2. Enter the data set in list L1 but without the outlier 19.0. What are the new mean, median, and standard deviation?
3. Did the mean or median change less when the outlier was deleted?

Check for Understanding

Concept Check

1. **OPEN ENDED** Give a sample set of data with a variance and standard deviation of 0.
2. **Find a counterexample** for the following statement.
   The standard deviation of a set of data is always less than the variance.
3. **Write** the formula for standard deviation using sigma notation. (*Hint:* To review sigma notation, see Lesson 11-5.)

Guided Practice

Find the variance and standard deviation of each set of data to the nearest tenth.

4. {48, 36, 40, 29, 45, 51, 38, 47, 39, 37}
5. {321, 322, 323, 324, 325, 326, 327, 328, 329, 330}
6. {43, 56, 78, 81, 47, 42, 34, 22, 78, 98, 38, 46, 54, 67, 58, 92, 55}

CONTENTS

Application

EDUCATION **For Exercises 7 and 8, use the following information.**
The table below shows the amounts of money spent on education per student in 1998 in two regions of the United States.

Pacific States		Southwest Central States	
State	Expenditures per Student ($)	State	Expenditures per Student ($)
Alaska	10,650	Texas	6291
California	5345	Arkansas	5222
Washington	6488	Louisiana	5194
Oregon	6719	Oklahoma	4634

Source: National Education Association

7. Find the mean for each region.
8. For which region is the mean more representative of the data? Explain.

Practice and Apply

Homework Help

For Exercises	See Examples
17–26	1
9–16, 27–33	2

Extra Practice

See page 855.

Find the variance and standard deviation of each set of data to the nearest tenth.

9. {400, 300, 325, 275, 425, 375, 350}
10. {5, 4, 5, 5, 5, 5, 6, 6, 6, 6, 7, 7, 7, 7, 8, 9}
11. {2.4, 5.6, 1.9, 7.1, 4.3, 2.7, 4.6, 1.8, 2.4}
12. {4.3, 6.4, 2.9, 3.1, 8.7, 2.8, 3.6, 1.9, 7.2}
13. {234, 345, 123, 368, 279, 876, 456, 235, 333, 444}
14. {13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 67, 56, 34, 99, 44, 55}

15.

Stem	Leaf
4	4 5 6 7 7
5	3 5 6 7 8 9
6	7 7 8 9 9 9

4|5 = 45

16.

Stem	Leaf
5	7 7 7 8 9
6	3 4 5 5 6 7
7	2 3 4 5 6

6|3 = 63

More About . . .

Basketball

Natalie Williams of the Utah Starzz led the Women's National Basketball Association in rebounding in 2000 with 336 rebounds in 29 games, an average of about 11.6 rebounds per game.

Source: WNBA

BASKETBALL **For Exercises 17 and 18, use the following information.**
The table below shows the rebounding totals for the 2000 Los Angeles Sparks.

306	179	205	194	105	55	122	32	23	16	23

Source: WNBA

17. Find the mean, median, and mode of the data to the nearest tenth.
18. Which measure of central tendency best represents the data? Explain.

Online Research **Data Update** For the latest rebounding statistics for both women's and men's professional basketball, visit: www.algebra2.com/data_update

EDUCATION **For Exercises 19 and 20, use the following information. The Millersburg school board is negotiating a pay raise with the teacher's union. Three of the administrators have salaries of $80,000 each. However, a majority of the teachers have salaries of about $35,000 per year.**

19. You are a member of the school board and would like to show that the current salaries are reasonable. Would you quote the mean, median, or mode as the "average" salary to justify your claim? Explain.
20. You are the head of the teacher's union and maintain that a pay raise is in order. Which of the mean, median, or mode would you quote to justify your claim? Explain your reasoning.

www.algebra2.com/self_check_quiz

ADVERTISING **For Exercises 21–23, use the following information.**
A camera store placed an ad in the newspaper showing five digital cameras for sale. The ad says, "Our digital cameras average $695." The prices of the digital cameras are $1200, $999, $1499, $895, $695, $1100, $1300, and $695.

21. Find the mean, median, and mode of the prices.

22. Which measure is the store using in its ad? Why did they choose this measure?

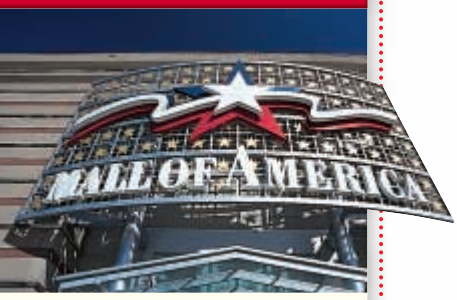
23. As a consumer, which measure would you want to see advertised? Explain.

SHOPPING MALLS **For Exercises 24–26, use the following information.**
The table lists the areas of some large shopping malls in the United States.

	Mall	Gross Leasable Area (ft^2)
1	Del Amo Fashion Center, Torrance, CA	3,000,000
2	South Coast Plaza/Crystal Court, Costa Mesa, CA	2,918,236
3	Mall of America, Bloomington, MN	2,472,500
4	Lakewood Center Mall, Lakewood, CA	2,390,000
5	Roosevelt Field Mall, Garden City, NY	2,300,000
6	Gurnee Mills, Gurnee, IL	2,200,000
7	The Galleria, Houston, TX	2,100,000
8	Randall Park Mall, North Randall, OH	2,097,416
9	Oakbrook Shopping Center, Oak Brook, IL	2,006,688
10	Sawgrass Mills, Sunrise, FL	2,000,000
10	The Woodlands Mall, The Woodlands, TX	2,000,000
10	Woodfield, Schaumburg, IL	2,000,000

Source: Blackburn Marketing Service

More About . . .



Shopping
While the Mall of America does not have the most gross leasable area, it is the largest fully enclosed retail and entertainment complex in the United States.
Source: Mall of America

24. Find the mean, median, and mode of the gross leasable areas.

25. You are a realtor who is trying to lease mall space in different areas of the country to a large retailer. Which measure would you talk about if the customer felt that the malls were too large for his store? Explain.

26. Which measure would you talk about if the customer had a large inventory? Explain.

FOOTBALL **For Exercises 27–30, use the weights in pounds of the starting offensive linemen of the football teams from three high schools.**

Jackson	Washington	King
170, 165, 140, 188, 195	144, 177, 215, 225, 197	166, 175, 196, 206, 219

27. Find the standard deviation of the weights for Jackson High.

28. Find the standard deviation of the weights for Washington High.

29. Find the standard deviation of the weights for King High

30. Which team had the most variation in weights? How do you think this variation will impact their play?

SCHOOL **For Exercises 31–33, use the frequency table at the right that shows the scores on a multiple-choice test.**

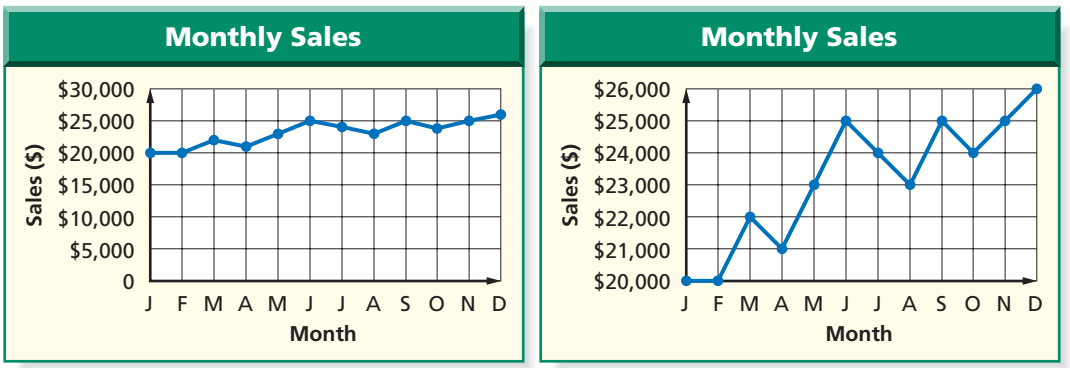
Score	Frequency
90	3
85	2
80	3
75	7
70	6
65	4

31. Find the variance and standard deviation of the scores.

32. What percent of the scores are within one standard deviation of the mean?

33. What percent of the scores are within two standard deviations of the mean?

For Exercises 34–36, consider the two graphs below.



34. Explain why the graphs made from the same data look different.

35. Describe a situation where the first graph might be used.

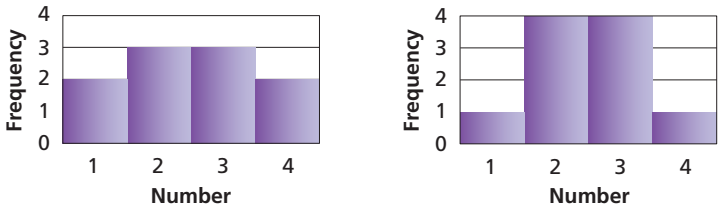
36. Describe a situation where the second graph might be used.

CRITICAL THINKING **For Exercises 37 and 38, consider the two sets of data.**

$$A = \{1, 2, 2, 2, 2, 3, 3, 3, 3, 4\},\ B = \{1, 1, 2, 2, 2, 3, 3, 3, 4, 4\}$$

37. Find the mean, median, variance, and standard deviation of each set of data to the nearest tenth.

38. Explain how you can tell which histogram below goes with each data set without counting the frequencies in the sets.



39. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

What statistics should a teacher tell the class after a test?

Include the following in your answer:

- the mean, median, and mode of the given data set, and
- which measure of central tendency you think best represents the test scores and why.

Standardized Test Practice

40. What is the mean of the numbers represented by $x + 1$, $3x - 2$, and $2x - 5$?

Ⓐ $2x - 2$ Ⓑ $\frac{6x - 7}{3}$ Ⓒ $\frac{x + 1}{3}$ Ⓓ $x + 4$

41. Manuel got scores of 92, 85, and 84 on three successive tests. What score must he get on a fourth test in order to have an average of 90?

Ⓐ 96 Ⓑ 97 Ⓒ 98 Ⓓ 99

Extending the Lesson

Mean deviation is another method of dispersion. It is the mean of the deviations of the data from the mean of the data. If a set of data consists of n values $x_1, x_2, \ldots, x_n$ and has mean $\bar{x}$, then the mean deviation is given by the following formula.

$$\text{MD} = \frac{|x_1 - \bar{x}| + |x_2 - \bar{x}| + \cdots + |x_n - \bar{x}|}{n} \text{ or } \frac{1}{n}\sum_{i=1}^{n} |x_i - \bar{x}|$$

Study Tip

Reading Math
Mean deviation is also sometimes called *mean absolute deviation*.

Find the mean deviation of each set of data to the nearest tenth.

42. {95, 91, 88, 86}

43. {10.4, 11.4, 16.2, 14.9, 13.5}

44. Suppose two sets of data have the same mean and different standard deviations. Describe their mean deviations.

Maintain Your Skills

Mixed Review

Determine whether the events are mutually exclusive or inclusive. Then find the probability. *(Lesson 12-5)*

45. A card is drawn from a standard deck of cards. What is the probability that it is a 5 or a spade?

46. A jar of change contains 5 quarters, 8 dimes, 10 nickels, and 19 pennies. If a coin is pulled from the jar at random, what is the probability that it is a nickel or a dime?

Two cards are drawn from a standard deck of cards. Find each probability. *(Lesson 12-4)*

47. P(ace, then king) if replacement occurs

48. P(ace, then king) if no replacement occurs

49. P(heart, then club) if no replacement occurs

50. P(heart, then club) if replacement occurs

51. Find the coordinates of the vertices and foci and the slopes of the asymptotes for the hyperbola given by $\frac{y^2}{81} - \frac{x^2}{25} = 1$. *(Lesson 8-5)*

If $f(x) = x - 7$, $g(x) = 4x^2$, and $h(x) = 2x + 1$, find each value. *(Lesson 7-7)*

52. $f[g(-1)]$ **53.** $h[f(15)]$ **54.** $f \circ h(2)$

55. **BUSINESS** The Energy Booster Company keeps their stock of Health Aid liquid in a rectangular tank whose sides measure $x - 1$ centimeters, $x + 3$ centimeters, and $x - 2$ centimeters. Suppose they would like to bottle their Health Aid in $x - 3$ containers of the same size. How much liquid in cubic centimeters will remain unbottled? *(Lesson 7-2)*

Use Cramer's Rule to solve each system of equations. *(Lesson 4-6)*

56. $2x + 6y = 28$
$-x - 4y = -20$

57. $7c - 3d = -8$
$4c + d = 9$

58. $m - 2n = -7$
$-3m + n = -4$

Getting Ready for the Next Lesson

BASIC SKILL **Find each percent.**

59. 68% of 200 **60.** 68% of 500 **61.** 95% of 400

62. 95% of 500 **63.** 99% of 400 **64.** 99% of 500

Practice Quiz 2

Lessons 12-4 through 12-6

A bag contains 5 red marbles, 3 green marbles, and 2 blue marbles. Two marbles are drawn at random from the bag. Find each probability. *(Lesson 12-4)*

1. P(red, then green) if replacement occurs **2.** P(red, then green) if no replacement occurs

3. P(2 red) if no replacement occurs **4.** P(2 red) if replacement occurs

A twelve-sided die has sides numbered 1 through 12. The die is rolled once. Find each probability. *(Lesson 12-5)*

5. P(4 or 5) **6.** P(even or a multiple of 3) **7.** P(odd or a multiple of 4)

Find the variance and standard deviation of each set of data to the nearest tenth. *(Lesson 12-6)*

8. {5, 8, 2, 9, 4} **9.** {16, 22, 18, 31, 25, 22} **10.** {425, 400, 395, 415, 420}

12-7 The Normal Distribution

What You'll Learn

- Determine whether a set of data appears to be normally distributed or skewed.
- Solve problems involving normally distributed data.

Vocabulary

- discrete probability distribution
- continuous probability distribution
- normal distribution
- skewed distribution

How are the heights of professional athletes distributed?

The frequency table below lists the heights of the 2001 Baltimore Ravens. The table shows the heights of the players, but it does not show how these heights compare to the height of an average player. To make that comparison, you can determine how the heights are distributed.

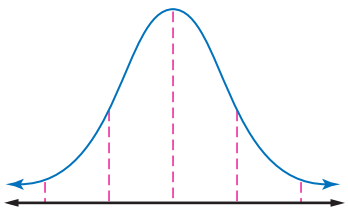
Height (in.)	67	69	70	71	72	73	74	75	76	77	80
Frequency	1	1	4	4	10	6	6	8	7	5	1

Source: www.ravenszone.net

NORMAL AND SKEWED DISTRIBUTIONS The probability distributions you have studied thus far are **discrete probability distributions** because they have only a finite number of possible values. A discrete probability distribution can be represented by a histogram. For a **continuous probability distribution**, the outcome can be any value in an interval of real numbers. Continuous probability distributions are represented by curves instead of histograms.

The curve at the right represents a continuous probability distribution. Notice that the curve is symmetric. Such a curve is often called a *bell curve*. Many distributions with symmetric curves or histograms are **normal distributions**.

Normal Distribution



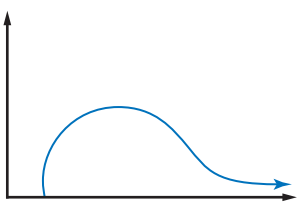
Study Tip

Skewed Distributions

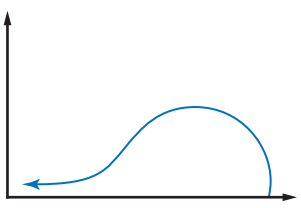
In a positively skewed distribution, the long tail is in the positive direction. These are sometimes said to be *skewed to the right*. In a negatively skewed distribution, the long tail is in the negative directon. These are sometimes said to be *skewed to the left*.

A curve or histogram that is not symmetric represents a **skewed distribution**. For example, the distribution for a curve that is high at the left and has a tail to the right is said to be *positively skewed*. Similarly, the distribution for a curve that is high at the right and has a tail to the left is said to be *negatively skewed*.

Positively Skewed



Negatively Skewed

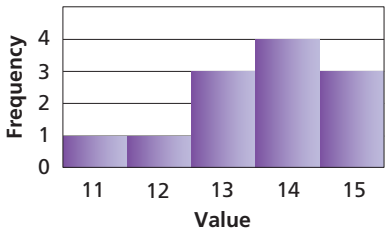


Example 1 Classify a Data Distribution

Determine whether the data {14, 15, 12, 11, 13, 13, 14, 15, 14, 12, 13, 14, 15} appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.

Make a frequency table for the data. Then use the table to make a histogram.

Value	11	12	13	14	15
Frequency	1	1	3	4	3



Since the histogram is high at the right and has a tail to the left, the data are negatively skewed.

USE NORMAL DISTRIBUTIONS

Normal distributions occur quite frequently in real life. Standardized test scores, the lengths of newborn babies, the useful life and size of manufactured items, and production levels can all be represented by normal distributions. In all of these cases, the number of data values must be large for the distribution to be approximately normal.

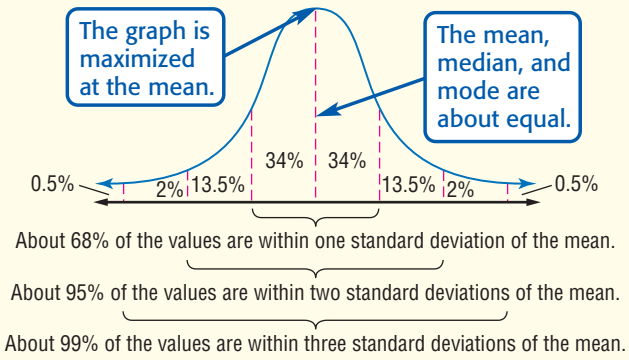
Study Tip

Normal Distribution

If you randomly select an item from data that are normally distributed, the probability that the one you pick will be within one standard deviation of the mean is 0.68. If you do this 1000 times, about 683 of those picked will be within one standard deviation of the mean.

Key Concept — Normal Distribution

Normal distributions have these properties.



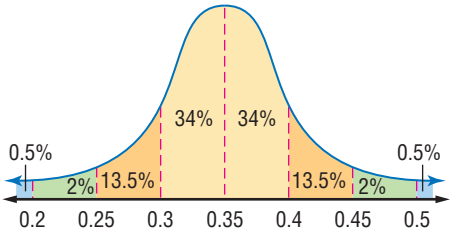
Example 2 Normal Distribution

PHYSIOLOGY **The reaction times for a hand-eye coordination test administered to 1800 teenagers are normally distributed with a mean of 0.35 second and a standard deviation of 0.05 second.**

a. About how many teens had reaction times between 0.25 and 0.45 second?

Draw a normal curve. Label the mean and the mean plus or minus multiples of the standard deviation.

The values 0.25 and 0.45 are 2 standard deviations *below and above* the mean, respectively. Therefore, about 95% of the data are between 0.25 and 0.45.



$1800 \times 95\% = 1710$ Multiply 1800 by 0.95.

About 1710 of the teenagers had reaction times between 0.25 and 0.45 second.

CONTENTS

b. What is the probability that a teenager selected at random had a reaction time greater than 0.4 second?

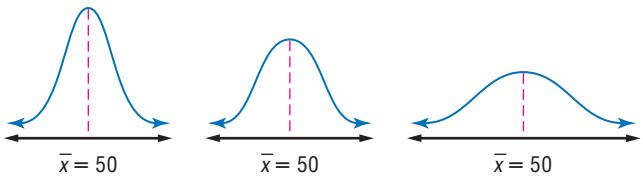
The value 0.4 is one standard deviation above the mean. You know that about 100% − 68% or 32% of the data are more than one standard deviation away from the mean. By the symmetry of the normal curve, half of 32%, or 16%, of the data are more than one standard deviation above the mean.

The probability that a teenager selected at random had a reaction time greater than 0.4 second is about 16% or 0.16.

Check for Understanding

Concept Check

1. **OPEN ENDED** Sketch a positively skewed graph. Describe a situation in which you would expect data to be distributed this way.
2. **Compare and contrast** the means and standard deviations of the graphs.



3. **Explain** how to find what percent of a set of normally distributed data is more than 3 standard deviations above the mean.

Guided Practice

4. The table at the right shows female mathematics SAT scores in 2000. Determine whether the data appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.

Score	Percent of Females
200–299	3
300–399	14
400–499	33
500–599	31
600–699	15
700–800	4

Source: www.collegeboard.org

For Exercises 5–7, use the following information.
Mrs. Sung gave a test in her trigonometry class. The scores were normally distributed with a mean of 85 and a standard deviation of 3.

5. What percent would you expect to score between 82 and 88?
6. What percent would you expect to score between 88 and 91?
7. What is the probability that a student chosen at random scored between 79 and 91?

Application

QUALITY CONTROL **For Exercises 8–11, use the following information.**
The useful life of a radial tire is normally distributed with a mean of 30,000 miles and a standard deviation of 5000 miles. The company makes 10,000 tires a month.

8. About how many tires will last between 25,000 and 35,000 miles?
9. About how many tires will last more than 40,000 miles?
10. About how many tires will last less than 25,000 miles?
11. What is the probability that if you buy a radial tire at random, it will last between 20,000 and 35,000 miles?

Practice and Apply

Homework Help

For Exercises	See Examples
12–14	1
15–26	2

Extra Practice

See page 856.

Determine whether the data in each table appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.

12.

U.S. Population	
Age	**Percent**
0–19	28.7
20–39	29.3
40–59	25.5
60–79	13.3
80–99	3.2
100+	0.0

Source: U.S. Census Bureau

13.

Record Low Temperatures in the 50 States	
Temperature (°F)	**Number of States**
−80 to −65	4
−64 to −49	12
−48 to −33	19
−32 to −17	12
−16 to −1	2
0 to 15	1

Source: *The World Almanac*

14. SCHOOL The frequency table at the right shows the grade-point averages (GPAs) of the juniors at Stanhope High School. Do the data appear to be *positively skewed*, *negatively skewed*, or *normally distributed*? Explain.

GPA	Frequency
0.0–0.4	4
0.5–0.9	4
1.0–1.4	2
1.5–1.9	32
2.0–2.4	96
2.5–2.9	91
3.0–3.4	110
3.5–4.0	75

FOOD **For Exercises 15–17, use the following information.**
The shelf life of a particular dairy product is normally distributed with a mean of 12 days and a standard deviation of 3.0 days.

15. About what percent of the products last between 9 and 15 days?

16. About what percent of the products last between 12 and 15 days?

17. About what percent of the products last less than 3 days?

18. About what percent of the products last more than 15 days?

VENDING **For Exercises 19–21, use the following information.**
The vending machine in the school cafeteria usually dispenses about 6 ounces of soft drink. Lately, it is not working properly, and the variability of how much of the soft drink it dispenses has been getting greater. The amounts are normally distributed with a standard deviation of 0.2 ounce.

19. What percent of the time will you get more than 6 ounces of soft drink?

20. What percent of the time will you get less than 6 ounces of soft drink?

21. What percent of the time will you get between 5.6 and 6.4 ounces of soft drink?

MANUFACTURING **For Exercises 22–24, use the following information.**
A company manufactures 1000 CDs per hour that are supposed to be 120 millimeters in diameter. These CDs are made for drives 122 millimeters wide. The sizes of CDs made by this company are normally distributed with a standard deviation of 1 millimeter.

22. What percent of the CDs would you expect to be greater than 120 millimeters?

23. In one hour, how many CDs would you expect to be between 119 and 122 millimeters?

24. About how many CDs per hour will be too large to fit in the drives?

More About . . .

Health

A systolic blood pressure below 130 is normal and between 130 and 139 is "high normal."

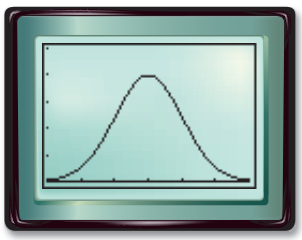
Source: National Institutes of Health

HEALTH **For Exercises 25 and 26, use the following information.**
A recent study showed that the systolic blood pressure of high school students ages 14–17 is normally distributed with a mean of 120 and a standard deviation of 12. Suppose a high school has 800 students.

25. About what percent of the students have blood pressures below 108?

26. About how many students have blood pressures between 108 and 144?

27. CRITICAL THINKING The graphing calculator screen shows the graph of a normal distribution for a large set of test scores whose mean is 500 and whose standard deviation is 100. If every test score in the data set were increased by 25 points, describe how the mean, standard deviation, and graph of the data would change.



[200, 800] scl: 100 by [0, 0.005] scl: 0.001

28. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are the heights of professional athletes distributed?

Include the following items in your answer:
- a histogram of the given data, and
- an explanation of whether you think the data are normally distributed.

Standardized Test Practice

A B C D

29. If $x + y = 5$ and $xy = 6$, what is the value of $x^2 + y^2$?

(A) 13 (B) 17 (C) 25 (D) 37

30. Which of the following is not the square of a rational number?

(A) 0.04 (B) 0.16 (C) $\frac{4}{9}$ (D) $\frac{2}{3}$

Maintain Your Skills

Mixed Review

Find the variance and standard deviation of each set of data to the nearest tenth. *(Lesson 12-6)*

31. {7, 16, 9, 4, 12, 3, 9, 4}

32. {12, 14, 28, 19, 11, 7, 10}

A card is drawn from a standard deck of cards. Find each probability. *(Lesson 12-5)*

33. P(jack or queen)

34. P(ace or heart)

35. P(2 or face card)

Find all of the rational zeros for each function. *(Lesson 7-6)*

36. $f(x) = x^3 + 4x^2 - 5x$

37. $p(x) = x^3 - 3x^2 - 10x + 24$

38. $h(x) = x^4 - 2x^2 + 1$

39. $f(x) = 4x^4 - 13x^3 - 13x^2 + 28x - 6$

METEOROLOGY **For excercises 40 and 41, use the following information.**
Weather forecasters can determine the approximate time that a thunderstorm will last if they know the diameter d of the storm in miles. The time t in hours can be found by using the formula $216t^2 = d^3$. *(Lesson 6-2)*

40. Graph $y = 216t^2 - 5^3$ and use it to estimate how long a thunderstorm will last if its diameter is 5 miles.

41. Find how long a thunderstorm will last if its diameter is 5 miles and compare this time with your estimate in Exercise 40.

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find the indicated term of each expression.**
*(For review of **binomial expansions**, see Lesson 5-2.)*

42. third term of $(a + b)^7$

43. fourth term of $(c + d)^8$

44. fifth term of $(x + y)^9$

www.algebra2.com/self_check_quiz

12-8 Binomial Experiments

What You'll Learn

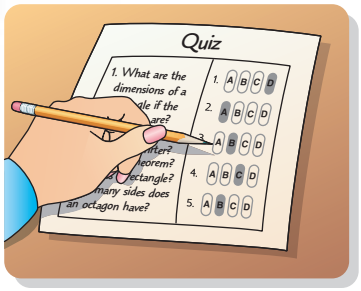
- Use binomial expansions to find probabilities.
- Find probabilities for binomial experiments.

Vocabulary

- binomial experiment

How can you determine whether guessing is worth it?

What is the probability of getting exactly 4 questions correct on a 5-question multiple-choice quiz if you guess at every question?



Study Tip

Look Back
To review the **Binomial Theorem**, see Lesson 11-7.

BINOMIAL EXPANSIONS

You can use the Binomial Theorem to find probabilities in certain situations where there are two possible outcomes. The 5 possible ways of getting 4 questions right r and 1 question wrong w are shown at the right. This chart shows the combination of 5 things (answer choices) taken 4 at a time (right answers) or $C(5, 4)$.

w	r	r	r	r
r	w	r	r	r
r	r	w	r	r
r	r	r	w	r
r	r	r	r	w

The terms of the binomial expansion of $(r + w)^5$ can be used to find the probabilities of each combination of right and wrong.

$$(r + w)^5 = r^5 + 5r^4w + 10r^3w^2 + 10r^2w^3 + 5rw^4 + w^5$$

Coefficient	Term	Meaning
$C(5, 5) = 1$	r^5	1 way to get all 5 questions right
$C(5, 4) = 5$	$5r^4w$	5 ways to get 4 questions right and 1 question wrong
$C(5, 3) = 10$	$10r^3w^2$	10 ways to get 3 questions right and 2 questions wrong
$C(5, 2) = 10$	$10r^2w^3$	10 ways to get 2 questions right and 3 questions wrong
$C(5, 1) = 5$	$5rw^4$	5 ways to get 1 question right and 4 questions wrong
$C(5, 0) = 1$	w^5	1 way to get all 5 questions wrong

The probability of getting a question right that you guessed on is $\frac{1}{4}$. So, the probability of getting the question wrong is $\frac{3}{4}$. To find the probability of getting 4 questions right and 1 question wrong, substitute $\frac{1}{4}$ for r and $\frac{3}{4}$ for w in the term $5r^4w$.

$$P(4 \text{ right, } 1 \text{ wrong}) = 5r^4w$$

$$= 5\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right) \quad r = \frac{1}{4},\ w = \frac{3}{4}$$

$$= \frac{15}{1024} \quad \text{Multiply.}$$

The probability of getting exactly 4 questions correct is $\frac{15}{1024}$ or about 1.5%.

CONTENTS

Example 1 Binomial Theorem

If a family has 4 children, what is the probability that they have 3 boys and 1 girl?

There are two possible outcomes for the gender of each of their children: boy or girl. The probability of a boy b is $\frac{1}{2}$, and the probability of a girl g is $\frac{1}{2}$.

$(b + g)^4 = b^4 + 4b^3g + 6b^2g^2 + 4bg^3 + g^4$

The term $4b^3g$ represents 3 boys and 1 girl.

$P(3 \text{ boys, } 1 \text{ girl}) = 4b^3g$

$= 4\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)$ $b = \frac{1}{2}, g = \frac{1}{2}$

$= \frac{1}{4}$ Multiply.

The probability of 3 boys and 1 girl is $\frac{1}{4}$ or 25%.

BINOMIAL EXPERIMENTS Problems like Example 1 that can be solved using binomial expansion are called **binomial experiments**.

Key Concept Binomial Experiments

A binomial experiment exists if and only if all of these conditions occur.

- There are exactly two possible outcomes for each trial.
- There is a fixed number of trials.
- The trials are independent.
- The probabilities for each trial are the same.

A binomial experiment is sometimes called a *Bernoulli experiment*.

Suppose that in the application at the beginning of the lesson, the first 3 questions are answered correctly. Then the last 2 are answered incorrectly. The probability of this occurring is $\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{3}{4} \cdot \frac{3}{4}$ or $\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2$. In general, there are $C(5, 3)$ ways to arrange 3 correct answers among the 5 questions, so the probability of exactly 3 correct answers is given by $C(5, 3)\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2$.

More About . . .

Sports

The National Hockey League record for most goals in a game by one player is seven. A player has scored five or more goals in a game 53 times in league history.

Source: NHL

Example 2 Binomial Experiment

SPORTS **Suppose that when hockey star Jaromir Jagr takes a shot, he has a $\frac{1}{7}$ probability of scoring a goal. He takes 6 shots in a game one night.**

a. What is the probability that he will score exactly 2 goals?

The probability that he scores a goal on a given shot is $\frac{1}{7}$. The probability that he does not score on a given shot is $\frac{6}{7}$. There are $C(6, 2)$ ways to choose the 2 shots that score.

$P(2 \text{ goals}) = C(6, 2)\left(\frac{1}{7}\right)^2\left(\frac{6}{7}\right)^4$ If he scores on 2 shots, he fails to score on 4 shots.

$= \frac{6 \cdot 5}{2}\left(\frac{1}{7}\right)^2\left(\frac{6}{7}\right)^4$ $C(6, 2) = \frac{6!}{4!2!}$

$= \frac{19{,}440}{117{,}649}$ Simplify.

The probability that Jagr will score exactly 2 goals is $\frac{19{,}440}{117{,}649}$ or about 0.17.

b. What is the probability that he will score at least 2 goals?

Instead of adding the probabilities of getting exactly 2, 3, 4, 5, and 6 goals, it is easier to subtract the probabilities of getting exactly 0 or 1 goal from 1.

$$\begin{aligned} P(\text{at least 2 goals}) &= 1 - P(0 \text{ goals}) - P(1 \text{ goal}) \\ &= 1 - C(6, 0)\left(\frac{1}{7}\right)^0\left(\frac{6}{7}\right)^6 - C(6, 1)\left(\frac{1}{7}\right)^1\left(\frac{6}{7}\right)^5 \\ &= 1 - \frac{46{,}656}{117{,}649} - \frac{46{,}656}{117{,}649} \quad \text{Simplify.} \\ &= \frac{24{,}337}{117{,}649} \quad \text{Subtract.} \end{aligned}$$

The probability that Jagr will score at least 2 goals is $\frac{24{,}337}{117{,}649}$ or about 0.21.

Check for Understanding

Concept Check

1. **OPEN ENDED** Describe a situation for which the P(2 or more) can be found by using a binomial expansion.
2. Refer to the application at the beginning of the lesson. List the possible sequences of 3 right answers and 2 wrong answers.
3. **Explain** why each experiment is not binomial.
   - **a.** rolling a die and recording whether a 1, 2, 3, 4, 5, or 6 comes up
   - **b.** tossing a coin repeatedly until it comes up heads
   - **c.** removing marbles from a bag and recording whether each one is black or white, if no replacement occurs

Guided Practice

Find each probability if a coin is tossed 3 times.

4. P(exactly 2 heads)
5. P(0 heads)
6. P(at least 1 head)

Four cards are drawn from a standard deck of cards. Each card is replaced before the next one is drawn. Find each probability.

7. P(4 jacks)
8. P(exactly 3 jacks)
9. P(at most 1 jack)

Application

SPORTS **For Exercises 10 and 11, use the following information.**
Jessica Mendoza of Stanford University was the 2000 NCAA women's softball batting leader with an average of .475. This means that the probability of her getting a hit in a given at-bat was 0.475.

10. Find the probability of her getting 4 hits in 4 at-bats.
11. Find the probability of her getting exactly 2 hits in 4 at-bats.

Practice and Apply

Homework Help

For Exercises	See Examples
12–37	1, 2

Extra Practice

See page 856.

Find each probability if a coin is tossed 4 times.

12. P(4 tails)
13. P(0 tails)
14. P(exactly 2 tails)
15. P(exactly 1 tail)
16. P(at least 3 tails)
17. P(at most 2 tails)

Find each probability if a die is rolled 5 times.

18. P(exactly one 5)
19. P(exactly three 5s)
20. P(at most two 5s)
21. P(at least three 5s)

CONTENTS

As an apartment manager, Jackie Thomas is responsible for showing prospective renters different models of apartments. When showing a model, the probability that she selects the correct key from her set is $\frac{1}{4}$. If she shows 5 models in a day, find each probability.

22. P(never the correct key)

23. P(always the correct key)

24. P(correct exactly 4 times)

25. P(correct exactly 2 times)

26. P(no more than 2 times correct)

27. P(at least 3 times correct)

Prisana guesses at all 10 true/false questions on her history test. Find each probability.

28. P(exactly 6 correct)

29. P(exactly 4 correct)

30. P(at most half correct)

31. P(at least half correct)

If a thumbtack is dropped, the probability of it landing point-up is 0.4. If 12 tacks are dropped, find each probability.

32. P(at least 9 points up)

33. P(at most 4 points up)

More About . . .

Internet

The word *Internet* was virtually unknown until the mid-1980s. By 1997, 19 million Americans were using the Internet. That number tripled in 1998 and passed 100 million in 1999.

Source: UCLA

34. CARS According to a recent survey, about 1 in 3 new cars is leased rather than bought. What is the probability that 3 of 7 randomly-selected new cars are leased?

35. INTERNET In 2001, it was estimated that 32.5% of U.S. adults use the Internet. What is the probability that exactly 2 out of 5 randomly-selected U.S. adults use the Internet?

WORLD CULTURES **For Exercises 36 and 37, use the following information.** The Cayuga Indians played a game of chance called *Dish*, in which they used 6 flattened peach stones blackened on one side. They placed the peach stones in a wooden bowl and tossed them. The winner was the first person to get a prearranged number of points. The table below shows the points that were given for each toss. Assume that each face (black or neutral) of each stone has an equal chance of showing up.

36. Copy and complete the table by finding the probability of each outcome.

37. Find the probability that a player gets at least 1 point for a toss.

Outcome	Points	Probability
6 black	5	
5 black, 1 neutral	1	
4 black, 2 neutral	0	
3 black, 3 neutral	0	
2 black, 4 neutral	0	
1 black, 5 neutral	1	
6 neutral	5	

38. CRITICAL THINKING Write an expression for the probability of exactly m successes in n trials of a binomial experiment where the probability of success in a given trial is p.

39. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How can you determine whether guessing is worth it?

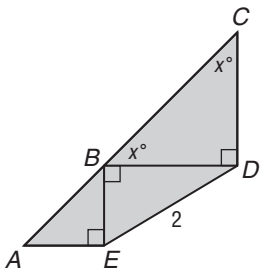
Include the following in your answer:

- an explanation of how to find the probability of getting any number of questions right on a 5-question multiple-choice quiz, and
- the probability of each score.

Standardized Test Practice

A B C D

40. GRID IN In the figure, if $DE = 2$, what is the sum of the area of $\triangle ABE$ and the area of $\triangle BCD$?



41. What is the net result if a discount of 5% is applied to a bill of \$340.60?

(A) \$306.54 (B) \$323.57

(C) \$335.60 (D) \$357.63

Graphing Calculator

BINOMIAL DISTRIBUTION You can use a TI-83 Plus to investigate the graph of a binomial distribution.

Step 1 Enter the number of trials in L1. Start with 10 trials.

KEYSTROKES: STAT 1 ▲ 2nd [LIST] ▶ 5 X,T,θ,*n* , X,T,θ,*n* , 0 , 10) ENTER

Step 2 Calculate the probability of success for each trial in L2.

KEYSTROKES: ▶ ▲ 2nd [DISTR] 0 10 , .5 , 2nd [L1]) ENTER

Step 3 Graph the histogram.

KEYSTROKES: 2nd [STATPLOT]

Use the arrow and ENTER keys to choose ON, the histogram, L1 as the Xlist, and L2 as the frequency. Use the window [0, 10] scl:1 by [0, 0.5] scl: 0.1.

42. Replace the 10 in the keystrokes for steps 1 and 2 to graph the binomial distribution for several values of n less than or equal to 47. You may have to adjust your viewing window to see all of the histogram. Make sure Xscl is 1.

43. What type of distribution does the binomial distribution start to resemble as n increases?

Maintain Your Skills

Mixed Review

For Exercises 44–46, use the following information.
A set of 400 test scores is normally distributed with a mean of 75 and a standard deviation of 8. *(Lesson 12-7)*

44. What percent of the test scores lie between 67 and 83?

45. How many of the test scores are greater than 91?

46. What is the probability that a randomly-selected score is less than 67?

47. A salesperson had sales of \$11,000, \$15,000, \$11,000, \$16,000, \$12,000, and \$12,000 in the last six months. Which measure of central tendency would he be likely to use to represent these data when he talks with his supervisor? Explain. *(Lesson 12-6)*

Graph each inequality. *(Lesson 2-7)*

48. $x \geq -3$ **49.** $x + y \leq 4$ **50.** $y > |5x|$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Evaluate $2\sqrt{\frac{p(1-p)}{n}}$ for the given values of p and n. Round to the nearest thousandth, if necessary.** *(For review of **radical expressions**, see Lesson 5-6.)*

51. $p = 0.5$, $n = 100$ **52.** $p = 0.5$, $n = 400$

53. $p = 0.25$, $n = 500$ **54.** $p = 0.75$, $n = 1000$

55. $p = 0.3$, $n = 500$ **56.** $p = 0.6$, $n = 1000$

Algebra Activity

A Follow-Up of Lesson 12-8

Simulations

A **simulation** uses a probability experiment to mimic a real-life situation. You can use a simulation to solve the following problem.

A brand of cereal is offering one of six different prizes in every box. If the prizes are equally and randomly distributed within the cereal boxes, how many boxes, on average, would you have to buy in order to get a complete set of the six prizes?

Collect the Data

Work in pairs or small groups to complete steps 1 through 4.

Step 1 Use the six numbers on a die to represent the six different prizes.

Step 2 Roll the die and record which prize was in the first box of cereal. Use a tally sheet like the one shown.

Step 3 Continue to roll the die and record the prize number until you have a complete set of prizes. Stop as soon as you have a complete set. This is the end of one trial in your simulation. Record the number of boxes required for this trial.

Step 4 Repeat steps 1, 2, and 3 until your group has carried out 25 trials. Use a new tally sheet for each trial.

Simulation Tally Sheet	
Prize Number	**Boxes Purchased**
1	
2	
3	
4	
5	
6	
Total Needed	

Analyze the Data

1. Create two different statistical graphs of the data collected for 25 trials.
2. Determine the mean, median, maximum, minimum, and standard deviation of the total number of boxes needed in the 25 trials.
3. Combine the small-group results and determine the mean, median, maximum, minimum, and standard deviation of the number of boxes required for all the trials conducted by the class.

Make a Conjecture

4. If you carry out 25 additional trials, will your results be the same as in the first 25 trials? Explain.
5. Should the small-group results or the class results give a better idea of the average number of boxes required to get a complete set of superheroes? Explain.
6. If there were 8 superheroes instead of 6, would you need to buy more boxes of cereal or fewer boxes of cereal on average?
7. What if one of the 6 prizes was more common than the other 5? For instance, suppose that one prize, Amazing Amy, appears in 25% of all the boxes and the other 5 prizes are equally and randomly distributed among the remaining 75% of the boxes? Design and carry out a new simulation to predict the average number of boxes you would need to buy to get a complete set. Include some measures of central tendency and dispersion with your data.

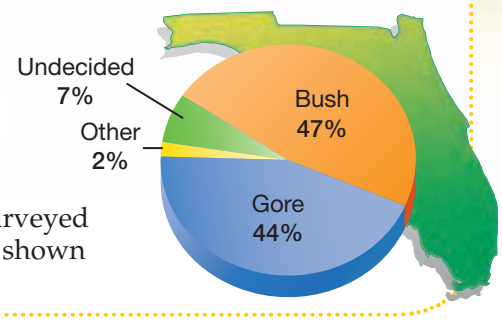
12-9 Sampling and Error

What You'll Learn

- Determine whether a sample is unbiased.
- Find margins of sampling error.

How are opinion polls used in political campaigns?

About a month before the 2000 presidential election, Mason-Dixon Polling & Research surveyed the preferences of Florida voters. The results shown were published in the *Orlando Sentinel*.



Vocabulary

- unbiased sample
- margin of sampling error

BIAS When polling organizations want to find how the public feels about an issue, they do not have the time or money to ask everyone. Instead, they obtain their results by polling a small portion of the population. To be sure that the results are representative of the population, they need to make sure that this portion is a random or **unbiased sample** of the population. A sample of size n is random when every possible sample of size n has an equal chance of being selected.

Example 1 Biased and Unbiased Samples

State whether each method would produce a random sample. Explain.

a. asking every tenth person coming out of a health club how many times a week they exercise to determine how often people in the city exercise

This would not result in a random sample because the people surveyed would probably exercise more often than the average person.

b. surveying people going into an Italian restaurant to find out people's favorite type of food

This would probably not result in a random sample because the people surveyed would probably be more likely than others to prefer Italian food.

MARGIN OF ERROR As the size of a sample increases, it more accurately reflects the population. If you sampled only three people and two prefer Brand A, you could say, "Two out of three people chose Brand A over any other brand," but you may not be giving a true picture of how the total population would respond. The **margin of sampling error (ME)** gives a limit on the difference between how a sample responds and how the total population would respond.

Key Concept Margin of Sampling Error

If the percent of people in a sample responding in a certain way is p and the size of the sample is n, then 95% of the time, the percent of the population responding in that same way will be between $p - ME$ and $p + ME$, where

$$ME = 2\sqrt{\frac{p(1-p)}{n}}.$$

That is, the probability is 0.95 that $p \pm ME$ will contain the true population results.

CONTENTS

Example 2 Find a Margin of Error

In a survey of 1000 randomly selected adults, 37% answered "yes" to a particular question. What is the margin of error?

$$ME = 2\sqrt{\frac{p(1-p)}{n}}$$ Formula for margin of sampling error

$$= 2\sqrt{\frac{0.37(1-0.37)}{1000}}$$ $p = 37\%$ or 0.37, $n = 1000$

$$\approx 0.030535$$ Use a calculator.

The margin of error is about 3%. This means that there is a 95% chance that the percent of people in the whole population who would answer "yes" is between 37 − 3 or 34% and 37 + 3 or 40%.

Published survey results often include the margin of error for the data. You can use this information to determinine the sample size.

Example 3 Analyze a Margin of Error

HEALTH In a recent Gallup Poll, 25% of the people surveyed said they had smoked cigarettes in the past week. The margin of error was 3%.

a. What does the 3% indicate about the results?

The 3% means that the probability is 95% that the percent of people in the population who had smoked cigarettes in the past week was between 25 − 3 or 22% and 25 + 3 or 28%.

b. How many people were surveyed?

$$ME = 2\sqrt{\frac{p(1-p)}{n}}$$ Formula for margin of sampling error

$$0.03 = 2\sqrt{\frac{0.25(1-0.25)}{n}}$$ $ME = 0.03$, p = 0.25

$$0.015 = \sqrt{\frac{0.25(0.75)}{n}}$$ Divide each side by 2.

$$0.000225 = \frac{0.25(0.75)}{n}$$ Square each side.

$$n = \frac{0.25(0.75)}{0.000225}$$ Multiply by n and divide by 0.000225.

$$n \approx 833.33$$ Use a calculator.

About 833 people were surveyed.

More About . . .

Health

The percent of smokers in the United States population declined from 38.7% in 1985 to 25.8% in 1999. New therapies, like the nicotine patch, are helping more people to quit.

Source: U.S. Department of Health and Human Services

Check for Understanding

Concept Check

1–3. See pp. 695A–695B.

1. **Describe** how sampling techniques can influence the results of a survey.
2. **OPEN ENDED** Give an example of a good sample and a bad sample. Explain your reasoning.
3. **Explain** what happens to the margin of sampling error when the size of the sample n increases. Why does this happen?

Guided Practice

Determine whether each situation would produce a random sample. Write *yes* or *no* and explain your answer.

4. the government sending a tax survey to everyone whose social security number ends in a particular digit

5. surveying students in the honors chemistry classes to determine the average time students in your school study each week

For Exercises 6–8, find the margin of sampling error to the nearest percent.

6. $p = 72\%$, $n = 100$ **7.** $p = 31\%$, $n = 500$

8. In a survey of 520 randomly-selected high school students, 68% of those surveyed stated that they were involved in extracurricular activities at their school.

Application

MEDIA **For Exercises 9 and 10, use the following information.**
According to a survey in *American Demographics*, 77% of Americans age 12 or older said they listen to the radio every day. Suppose the survey had a margin of error of 5%.

9. What does the 5% indicate about the results?

10. How many people were surveyed?

Practice and Apply

Homework Help

For Exercises	See Examples
11–14	1
15–26	2
27, 28	3

Extra Practice

See page 856.

Determine whether each situation would produce a random sample. Write *yes* or *no* and explain your answer.

11. pointing with your pencil at a class list with your eyes closed as a way to find a sample of students in your class

12. putting the names of all seniors in a hat, then drawing names from the hat to select a sample of seniors

13. calling every twentieth person listed in the telephone book to determine which political candidate is favored

14. finding the heights of all the boys in a freshman physical education class to determine the average height of all the boys in your school

For Exercises 15–24, find the margin of sampling error to the nearest percent.

15. $p = 81\%$, $n = 100$ **16.** $p = 16\%$, $n = 400$ **17.** $p = 54\%$, $n = 500$

18. $p = 48\%$, $n = 1000$ **19.** $p = 33\%$, $n = 1000$ **20.** $p = 67\%$, $n = 1500$

21. A poll asked people to name the most serious problem facing the country. Forty-six percent of the 800 randomly selected people said crime.

22. Although skim milk has as much calcium as whole milk, only 33% of 2406 adults surveyed in *Shape* magazine said skim milk is a good calcium source.

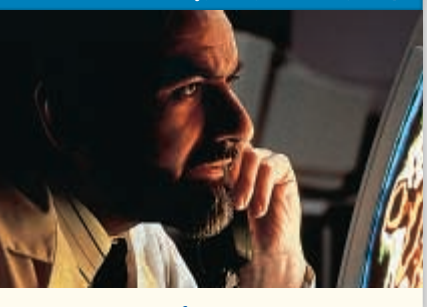
23. Three hundred sixty-seven of 425 high school students said pizza was their favorite food in the school cafeteria.

24. Nine hundred thirty-four of 2150 subscribers to a particular newspaper said their favorite sport was football.

25. **ECONOMICS** In a poll conducted by ABC News, 83% of the 1020 people surveyed said they supported raising the minimum wage. What was the margin of error?

CONTENTS

Career Choices



Physican

Physicians diagnose illnesses and prescribe and administer treatment.

Online Research

For information about a career as a physician, visit: www.algebra2.com/careers

26. PHYSICIANS In a recent Harris Poll, 61% of the 1010 people surveyed said they considered being a physician to be a very prestigious occupation. What was the margin of error? **about 3%**

27. SHOPPING According to a Gallup Poll, 33% of shoppers planned to spend $1000 or more during a recent holiday season. The margin of error was 3%. How many people were surveyed? **about 983**

28. CRITICAL THINKING One hundred people were asked a yes-or-no question in an opinion poll. How many said "yes" if the margin of error was 9.6%? **36 or 64**

29. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. **See pp. 695A–695B.**

How are opinion polls used in political campaigns?

Include the following in your answer:

- a description of how a candidate could use statistics from opinion polls to determine where to make campaign stops,
- the margin of error for Bush if 807 people were surveyed, and
- an explanation of how to use the margin of error to determine the range of percent of Florida voters who favored Bush.

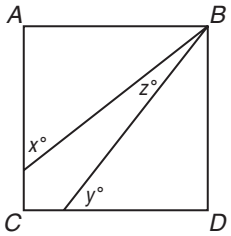
Standardized Test Practice

A B C D

30. In rectangle $ABCD$, what is $x + y$ in terms of z? **A**

(A) $90 + z$ (B) $190 - z$

(C) $180 + z$ (D) $270 - z$



31. If $xy^{-2} + y^{-1} = y^{-2}$, then the value of x *cannot* equal which of the following? **C**

(A) -1 (B) 0 (C) 1 (D) 2

Maintain Your Skills

Mixed Review

A student guesses at all 5 questions on a true-false quiz. Find each probability. *(Lesson 12-8)*

32. P(all 5 correct) $\frac{1}{32}$ **33.** P(exactly 4 correct) $\frac{5}{32}$ **34.** P(at least 3 correct) $\frac{1}{2}$

A set of 250 data values is normally distributed with a mean of 50 and a standard deviation of 5.5. *(Lesson 12-7)*

35. What percent of the data lies between 39 and 61? **95%**

36. How many data values are less than 55.5? **210**

37. What is the probability that a data value selected at random is greater than 39? **37. 97.5%**

38. Given $x^3 - 3x^2 - 4x + 12$ and one of its factors $x + 2$, find the remaining factors of the polynomial. *(Lesson 7-4)* **$x - 2$, $x - 3$**

WebQuest Internet Project

'Minesweeper': Secret to Age-Old Puzzle?

It is time to complete your project. Use the information and data you have gathered about the history of mathematics to prepare a presentation or web page. Be sure to include transparencies and a sample mathematics problem or idea in the presentation.

www.algebra2.com/webquest

www.algebra2.com/extra_examples

Algebra Activity

A Follow-Up of Lesson 12-9

Testing Hypotheses

A **hypothesis** is a statement to be tested. Testing a hypothesis to determine whether it is supported by the data involves five steps.

Step 1 State the hypothesis. The statement should include a *null hypothesis*, which is the hypothesis to be tested, and an *alternative hypothesis*.

Step 2 Design the experiment.

Step 3 Conduct the experiment and collect the data.

Step 4 Evaluate the data. Decide whether to reject the null hypothesis.

Step 5 Summarize the results.

Test the following hypothesis.

People react to sound and touch at the same rate.

You can measure reaction time by having someone drop a ruler and then having someone else catch it between their fingers. The distance the ruler falls will depend on their reaction time. Half of the class will investigate the time it takes to react when someone is told the ruler has dropped. The other half will measure the time it takes to react when the catcher is alerted by touch.

Step 1 The null hypothesis H_0 and alternative hypothesis H_1 are as follows.
These statements often use $=$, $\neq$, $<$, $>$, $\geq$, and $\leq$.

- H_0: reaction time to sound $=$ reaction time to touch
- H_1: reaction time to sound $\neq$ reaction time to touch

Step 2 You will need to decide the height from which the ruler is dropped, the position of the person catching the ruler, the number of practice runs, and whether to use one try or the average of several tries.

Step 3 Conduct the experiment in each group and record the results.

Step 4 Organize the results so that they can be compared.

Step 5 Based on the results of your experiment, do you think the hypothesis is true? Explain.

Analyze

State the null and alternative hypotheses for each conjecture. **1–3. See pp. 695A–695B.**

1. A teacher feels that playing classical music during a math test will cause the test scores to change (either up or down). In the past, the average test score was 73.

2. An engineer thinks that the mean number of defects can be decreased by using robots on an assembly line. Currently, there are 18 defects for every 1000 items.

3. A researcher is concerned that a new medicine will cause pulse rates to rise dangerously. The mean pulse rate for the population is 82 beats per minute.

4. MAKE A CONJECTURE Design an experiment to test the following hypothesis.
Pulse rates increase 20% after moderate exercise. **See students' work.**

Chapter 12

Study Guide and Review

Vocabulary and Concept Check

area diagram (p. 651)
binomial experiment (p. 677)
combination (p. 640)
compound event (p. 658)
continuous probability distribution (p. 671)
dependent events (p. 633)
discrete probability distributions (p. 671)
event (p. 632)
failure (p. 644)
Fundamental Counting Principle (p. 633)
inclusive events (p. 659)
independent events (p. 632)
linear permutation (p. 638)
margin of sampling error (p. 682)
measure of central tendency (p. 664)
measure of variation (p. 665)
mutually exclusive events (p. 658)
normal distribution (p. 671)
odds (p. 645)
outcome (p. 632)
permutation (p. 638)
probability (p. 644)
probability distribution (p. 646)
random (p. 646)
random variable (p. 645)
relative-frequency histogram (p. 646)
sample space (p. 632)
simple event (p. 658)
skewed distribution (p. 671)
standard deviation (p. 665)
success (p. 644)
unbiased sample (p. 682)
variance (p. 665)

Choose the letter of the term that best matches each statement or phrase.

1. the ratio of the number of ways an event can succeed to the number of possible outcomes
2. an arrangement of objects in which order does not matter
3. two or more events in which the outcome of one event affects the outcome of another event
4. a sample in which every member of the population has an equal chance to be selected
5. an arrangement of objects in which order matters
6. two events in which the outcome can never be the same
7. the ratio of the number of ways an event can succeed to the number of ways it can fail

a. dependent events
b. combination
c. probability
d. permutation
e. mutually exclusive events
f. odds
g. unbiased sample

Lesson-by-Lesson Review

12-1 The Counting Principle

See pages 632–637.

Concept Summary

- Fundamental Counting Principle: If event M can occur in m ways and is followed by event N that can occur in n ways, then the event M followed by the event N can occur in $m \cdot n$ ways.
- Independent Events: The outcome of one event does *not* affect the outcome of another.
- Dependent Events: The outcome of one event *does* affect the outcome of another.

Example

How many different license plates are possible with two letters followed by three digits?

There are 26 possibilities for each letter. There are 10 possibilities, the digits 0–9, for each number. Thus, the number of possible license plates is as follows.

$26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 26^2 \cdot 10^3$ or 676,000

Exercises **Solve each problem.** *See Examples 2 and 3 on page 633.*

8. The letters a, c, e, g, i, and k are used to form 6-letter passwords for a movie theater security system. How many passwords can be formed if the letters can be used more than once in any given password?
9. How many 4-digit personal identification codes can be formed if each numeral can only be used once?

12-2 Permutations and Combinations

See pages 638–643.

Concept Summary

- In a permutation, the order of objects is important.
- In a combination, the order of objects is not important.

Example

A basket contains 3 apples, 6 oranges, 7 pears, and 9 peaches. How many ways can 1 apple, 2 oranges, 6 pears, and 2 peaches be selected?

This involves the product of four combinations, one for each type of fruit.

$$C(3, 1) \cdot C(6, 2) \cdot C(7, 6) \cdot C(9, 2) = \frac{3!}{(3-1)!1!} \cdot \frac{6!}{(6-2)!2!} \cdot \frac{7!}{(7-6)!6!} \cdot \frac{9!}{(9-2)!2!}$$
$$= 3 \cdot 15 \cdot 7 \cdot 36 \text{ or } 11{,}340$$

There are 11,340 different ways to choose the fruit from the basket.

Exercises **Solve each problem.** *See Example 4 on page 640.*

10. A committee of 3 is selected from Jillian, Miles, Mark, and Nikia. How many committees contain 2 boys and 1 girl?
11. Five cards are drawn from a standard deck of cards. How many different hands consist of four queens and one king?
12. A box of pencils contains 4 red, 2 white, and 3 blue pencils. How many different ways can 2 red, 1 white, and 1 blue pencil be selected?

12-3 Probability

See pages 644–650.

Concept Summary

- $P(\text{success}) = \frac{s}{s+f}$; $P(\text{failure}) = \frac{f}{s+f}$
- odds of success $= s{:}f$; odds of failure $= f{:}s$

Example

A bag of golf tees contains 23 red, 19 blue, 16 yellow, 21 green, 11 orange, 19 white, and 17 black tees. What is the probability that if you choose a tee from the bag at random, you will choose a green tee?

There are 21 ways to choose a green tee and 23 + 19 + 16 + 11 + 19 + 17 or 105 ways not to choose a green tee. So, s is 21 and f is 105.

$$P(\text{green tee}) = \frac{s}{s+f}$$
$$= \frac{21}{21+105} \text{ or } \frac{1}{6}$$

The probability is 1 out of 6 or about 16.7%.

CONTENTS

Exercises **Find the odds of an event occurring, given the probability of the event.**
See Example 3 on pages 645 and 646.

13. $\frac{1}{4}$ **14.** $\frac{5}{8}$ **15.** $\frac{7}{12}$ **16.** $\frac{3}{7}$ **17.** $\frac{2}{5}$

18. The table shows the distribution of the number of heads occurring when four coins are tossed. Find $P(H = 3)$.
See Example 4 on page 646.

H = Heads	0	1	2	3	4
Probability	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

12-4 Multiplying Probabilities

See pages 651–657.

Concept Summary

- Probability of two independent events: $P(A \text{ and } B) = P(A) \cdot P(B)$
- Probability of two dependent events: $P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$

Example

There are 3 dimes, 2 quarters, and 5 nickels in Langston's pocket. If he reaches in and selects three coins at random without replacing any of them, what is the probability that he will choose a dime d, then a quarter q, then a nickel n?

Because the outcomes of the first and second choices affect the later choices, these are dependent events.

$P(d, \text{ then } q, \text{ then } n) = \frac{3}{10} \cdot \frac{2}{9} \cdot \frac{5}{8}$ or $\frac{1}{24}$ The probability is $\frac{1}{24}$ or about 4.2%.

Exercises **Determine whether the events are *independent* or *dependent*. Then find the probability.** *See Examples 1–4 on pages 652 and 654.*

19. Two dice are rolled. What is the probability that each die shows a 4?

20. Two cards are drawn from a standard deck of cards without replacement. Find the probability of drawing a heart and a club, in that order.

21. Luz has 2 red, 2 white, and 3 blue marbles in a cup. If she draws two marbles at random and does not replace the first one, find the probability of a white marble and then a blue marble.

12-5 Adding Probabilities

See pages 658–663.

Concept Summary

- Probability of mutually exclusive events: $P(A \text{ or } B) = P(A) + P(B)$
- Probability of inclusive events: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Example

Trish has four \$1 bills and six \$5 bills. She takes three bills from her wallet at random. What is the probability that Trish will select at least two \$1 bills?

$$P(\text{at least two \$1 bills}) = P(\text{two \$1, one \$5}) + P(\text{three \$1, no \$5})$$

$$= \frac{C(4, 2) \cdot C(6, 1)}{C(10, 3)} + \frac{C(4, 3) \cdot C(6, 0)}{C(10, 3)}$$

$$= \frac{4! \cdot 6!}{(4-2)!2!(6-1)!1!} + \frac{4! \cdot 6!}{(4-3)!3!(6-0)!0!}$$

$$= \frac{36}{120} + \frac{4}{120} \text{ or } \frac{1}{3}$$ The probability is $\frac{1}{3}$ or about 0.333.

Exercises **Determine whether the events are *mutually exclusive* or *inclusive*. Then find the probability.** *See Examples 1–3 on pages 659 and 660.*

22. There are 5 English, 2 math, and 3 chemistry books on a shelf. If a book is randomly selected, what is the probability of selecting a math book or a chemistry book? **mutually exclusive; $\frac{1}{2}$**

23. A die is rolled. What is the probability of rolling a 6 or a number less than 4? **23. mutually exclusive; $\frac{2}{3}$**

24. A die is rolled. What is the probability of rolling a 6 or a number greater than 4?

25. A card is drawn from a standard deck of cards. What is the probability of drawing a king or a red card? **inclusive; $\frac{7}{13}$**

24. inclusive; $\frac{1}{3}$

12-6 Statistical Measures

See pages 664–670.

Concept Summary

- To represent a set of data, use the mean if the data are spread out and you want an average of the values, the median when the data contain outliers, or the mode when the data are tightly clustered around one or two values.
- Standard deviation for n values:

$$\sigma = \sqrt{\frac{(x^1 - \bar{x})^2 + (x^2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n}}, \bar{x} \text{ is the mean}$$

Example

Find the variance and standard deviation for {100, 156, 158, 159, 162, 165, 170, 190}.

Step 1 Find the mean.

$$\bar{x} = \frac{100 + 156 + 158 + 159 + 162 + 165 + 170 + 190}{8}$$ Add the data and divide by the number of items.

$$= \frac{1260}{8}$$

$$= 157.5$$

Step 2 Find the variance.

$$\sigma^2 = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n}$$

$$= \frac{(100 - 157.5)^2 + (156 - 157.5)^2 + \cdots + (170 - 157.5)^2 + (190 - 157.5)^2}{8}$$

$$= \frac{4600}{8}$$ Simplify.

$$= 575$$ Use a calculator.

Step 3 Find the standard deviation.

$\sigma^2 = 575$ Take the square root of each side.

$\sigma \approx 23.98$ Use a calculator.

Exercises **Find the variance and standard deviation of each set of data to the nearest tenth.** *See Examples 1 and 2 on pages 664 and 665.*

26. {56, 56, 57, 58, 58, 58, 59, 61} **2.4, 1.5**

27. {302, 310, 331, 298, 348, 305, 314, 284, 321, 337} **341.0, 18.5**

28. {3.4, 4.2, 8.6, 5.1, 3.6, 2.8, 7.1, 4.4, 5.2, 5.6} **2.8, 1.7**

CONTENTS

12-7 The Normal Distribution

See pages 671–675.

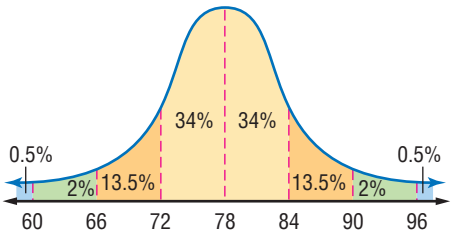
Concept Summary

Normal distributions have these properties.

- The graph is maximized and the data are symmetric at the mean.
- The mean, median, and mode are about equal.
- About 68% of the values are within one standard deviation of the mean.
- About 95% of the values are within two standard deviations of the mean.
- About 99% of the values are within three standard deviations of the mean.

Example

Mr. Byrum gave an exam to his 30 Algebra 2 students at the end of the first semester. The scores were normally distributed with a mean score of 78 and a standard deviation of 6.



a. What percent of the class would you expect to have scored between 72 and 84?

Since 72 and 84 are 1 standard deviation to the left and right of the mean, respectively, 34% + 34% or 68% of the students scored within this range.

b. What percent of the class would you expect to have scored between 90 and 96?

90 to 96 on the test includes 2% of the students.

c. Approximately how many students scored between 84 and 90?

84 to 90 on the test includes 13.5% of the students. $0.135 \times 30 = 4$ students

d. Approximately how many students scored between 72 and 84?

34% + 34% or 68% of the students scored between 72 and 84.

$0.68 \times 30 = 20$ students

Exercises **For Exercises 29–32, use the following information.**

The utility bills in a city of 5000 households are normally distributed with a mean of $180 and a standard deviation of $16. *See Example 2 on pages 672 and 673.*

29. About how many utility bills were between $164 and $196? **3400**

30. About how many bills were more than $212? **125**

31. About how many bills were less than $164? **800**

32. What is the probability that a household selected at random will have a utility bill between $164 and $180? **34%**

12-8 Binomial Experiments

See pages 676–680.

Concept Summary

A binomial experiment exists if and only if all of these conditions occur.

- There are exactly two possible outcomes for each trial.
- There is a fixed number of trials.
- The trials are independent.
- The possibilities for each trial are the same.

Example

To practice for a jigsaw puzzle competition, Laura and Julian completed four jigsaw puzzles. The probability that Laura places the last piece is $\frac{3}{5}$, and the probability that Julian places the last piece is $\frac{2}{5}$. What is the probability that Laura will place the last piece of at least two puzzles?

$P = L^4 + 4L^3J + 6L^2J^2$ — P(last piece in 4) + P(last piece in 3) + P(last piece in 2)

$= \left(\frac{3}{5}\right)^4 + 4\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right) + 6\left(\frac{3}{5}\right)^2\left(\frac{2}{5}\right)^2$ — $L = \frac{3}{5}, J = \frac{2}{5}$

$= \frac{81}{625} + \frac{216}{625} + \frac{216}{625}$ or 0.8208 — The probability is 82.08%.

Exercises *See Example 2 on pages 677 and 678.*

33. Find the probability of getting 7 heads in 8 tosses of a coin.

34. Find the probability that a family with seven children has exactly five boys.

Find each probability if a die is rolled twelve times.

35. P(twelve 3s) **36.** P(exactly one 3) **37.** P(six 3s)

12-9 Sampling and Error

See pages 682–685.

Concept Summary

- Margin of sampling error: $ME = 2\sqrt{\frac{p(1-p)}{n}}$ if the percent of people in a sample responding in a certain way is p and the size of the sample is n

Example

In a survey taken at a local high school, 75% of the student body stated that they thought school lunches should be free. This survey had a margin of error of 2%. How many people were surveyed?

$ME = 2\sqrt{\frac{p(1-p)}{n}}$ — Formula for margin of sampling error

$0.02 = 2\sqrt{\frac{0.75(1-0.75)}{n}}$ — $ME = 0.02$, $p = 0.75$

$0.01 = \sqrt{\frac{0.75(1-0.75)}{n}}$ — Divide each side by 2.

$0.0001 = \frac{0.75(0.25)}{n}$ — Square each side of the equation.

$n = \frac{0.75(0.25)}{0.0001}$ — Multiply each side by n and divide each side by 0.0001.

$n = 1875$ — There were about 1875 people in the survey.

Exercises

38. In a poll asking people to name their most valued freedom, 51% of the randomly selected people said it was the freedom of speech. Find the margin of sampling error if 625 people were randomly selected. *See Example 2 on page 683.*

39. According to a recent survey of mothers with children who play sports, 63% of them would prefer that their children not play football. Suppose the margin of error is 4.5%. How many mothers were surveyed? *See Example 3 on page 683.*

Chapter 12

Practice Test

Vocabulary and Concepts

Match the following terms and descriptions.

1. data are symmetric about the mean
2. variance and standard deviation
3. mode, median, mean

a. measures of central tendency
b. measures of variation
c. normal distribution

Skills and Applications

Evaluate each expression.

4. $P(7, 3)$
5. $C(7, 3)$
6. $P(13, 5)$

Solve each problem.

7. How many ways can 9 bowling balls be arranged on the upper rack of a bowling ball rack?
8. How many different outfits can be made if you choose 1 each from 11 skirts, 9 blouses, 3 belts, and 7 pairs of shoes?
9. How many ways can the letters of the word *probability* be arranged?
10. How many different soccer teams consisting of 11 players can be formed from 18 players?
11. In a row of 10 parking spaces in a parking lot, how many ways can 4 cars park?
12. Eleven points are equally spaced on a circle. How many ways can 5 of these points be chosen as the vertices of a pentagon?
13. A number is drawn at random from a hat that contains all the numbers from 1 to 100. What is the probability that the number is less than sixteen?
14. Two cards are drawn in succession from a standard deck of cards without replacement. What is the probability that both cards are greater than 2 and less than 9?
15. A shipment of ten television sets contains 3 defective sets. How many ways can a hospital purchase 4 of these sets and receive at least 2 of the defective sets?
16. While shooting arrows, William Tell can hit an apple 9 out of 10 times. What is the probability that he will hit it exactly 4 out of 7 times?
17. Ten people are going on a camping trip in 3 cars that hold 5, 2, and 4 passengers, respectively. How many ways is it possible to transport the 10 people to their campsite?
18. From a box containing 5 white golf balls and 3 red golf balls, 3 golf balls are drawn in succession, each being replaced in the box before the next draw is made. What is the probability that all 3 golf balls are the same color?

For Exercises 19–21, use the following information.
In a ten-question multiple-choice test with four choices for each question, a student who was not prepared guesses on each item. Find each probability.

19. six questions correct
20. at least eight questions correct
21. fewer than eight questions correct

22. **STANDARDIZED TEST PRACTICE** Lila throws a die and writes down the number showing. If she throws the number cube again, what is the probability that the second throw will have the same number showing as the first throw?

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{1}{6}$

Chapter 12 Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

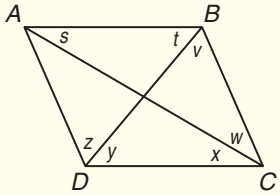
1. In a jar of red and green gumdrops, the ratio of red gumdrops to green gumdrops is 7 to 3. If the jar contains a total of 150 gumdrops, how many gumdrops are green?

Ⓐ 21 Ⓑ 30
Ⓒ 45 Ⓓ 105

2. $\langle x \rangle = \frac{1}{2}x$ if x is composite and $\langle x \rangle = 2x$ if x is prime. What is the value of $\langle 16 \rangle + \langle 11 \rangle$?

Ⓐ 10 Ⓑ 30
Ⓒ 54 Ⓓ 60

3. In rhombus $ABCD$, which of the following are true?



I. $\angle s$ and $\angle x$ are congruent.

II. $\angle t$ and $\angle v$ are congruent.

III. $\angle z$ and $\angle t$ are congruent.

Ⓐ I only
Ⓑ II only
Ⓒ I and II only
Ⓓ I, II, and III

4. What is the area of an isosceles right triangle with hypotenuse $3\sqrt{2}$ units?

Ⓐ $1.5\sqrt{2}$ units^2
Ⓑ 4.5 units^2
Ⓒ 9 units^2
Ⓓ $6 + 3\sqrt{2}$ units^2

5. What is the solution set for $t(t + 7) = 18$?

Ⓐ $\{-2, 9\}$
Ⓑ $\{-3, 6\}$
Ⓒ $\{0, 18\}$
Ⓓ $\{-9, 2\}$

6. The equation $3x - 8 = 5x^2 - y$ represents which of the following conic sections?

Ⓐ hyperbola
Ⓑ parabola
Ⓒ circle
Ⓓ ellipse

7. If the equations $x^2 + y^2 = 16$ and $y = x^2 + 4$ are graphed on the same coordinate plane, how many points of intersection exist?

Ⓐ none
Ⓑ one
Ⓒ two
Ⓓ three

8. A number is chosen at random from the set $\{1, 2, 3, \ldots 20\}$. What is the probability that the number is odd and divisible by 3?

Ⓐ $\frac{3}{20}$ Ⓑ $\frac{3}{10}$
Ⓒ $\frac{7}{20}$ Ⓓ $\frac{13}{20}$

9. What is the least positive integer that is divisible by 3, 4, 5, and 6?

Ⓐ 60 Ⓑ 180
Ⓒ 240 Ⓓ 360

10. If $4y - 5x + 6xy - 50 = 0$ and $x + 7 = 13$, then what is $y + 5$?

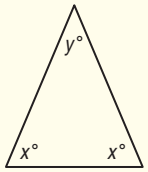
Ⓐ 2 Ⓑ 6
Ⓒ 7 Ⓓ 11

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

11. In a high school, 250 students take math and 50 students take art. If there are 280 students enrolled in the school and they all take at least one of these courses, how many students take both math and art?

12. If $20 < y < 30$ and x and y are both integers, what is the greatest possible value for x?



13. Four numbers are selected at random. Their average (arithmetic mean) is 45. The fourth number selected is 34. What is the sum of the other three numbers?

14. If one half of an even positive integer and three fourths of the next greater even integer have a sum of 24, what is the mean of the two integers?

15. Shane has six tiles, each of which has one of the letters A, B, C, D, E, or F on it. If one of the letters must be A and the last letter must be F, how many different arrangements of three letters (such as ADF) can Shane create with these titles?

The Princeton Review Test-Taking Tip

Question 14 When answering short-response questions, read carefully and make sure that you know exactly what the question is asking you to find. For example, if you only find the value of y in Question 14, you have not solved the problem. You need to find the value of $y + 5$.

Part 3 Quantitative Comparison

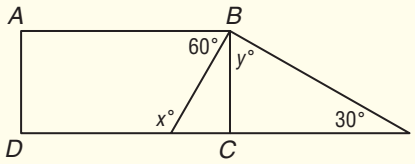
Compare the quantity in Column A and the quantity in Column B. Then determine whether:

- **(A) the quantity in Column A is greater,**
- **(B) the quantity in Column B is greater,**
- **(C) the two quantities are equal, or**
- **(D) the relationship cannot be determined from the information given.**

16. $x < 0$

Column A	Column B
$x - 2$	$2 - x$

17. $ABCD$ is a rectangle.



Column A	Column B
$(x + y)°$	$180°$

18. $x > y$, $w < z$
w, x, y, and z are positive integers.

Column A	Column B
$\frac{y}{z}$	$\frac{x}{w}$

19. For $t \neq 0$, $\langle t \rangle = \frac{t^2 - 1}{t}$.

Column A	Column B
$\langle 2 \rangle$	$\langle -2 \rangle$

20. For $t \neq 0$, $\langle t \rangle = \frac{t^2 - 1}{t}$.

Column A	Column B
$\langle 1 \rangle$	$\langle -1 \rangle$

21. $y = -3$

Column A	Column B
y^2	y^{-2}